Hydro-electric Unit Commitment Subject to Uncertain Demand

Hamish Waterer
Department of Engineering Science
University of Auckland
New Zealand
waterer@esvl.auckland.ac.nz

Abstract
Hydro-electric power utilities convert the potential energy of water to electricity through the operation of generation turbines. Each turbine unit has an operating range and incurs a fixed start-up cost. The problem of determining which turbines should be operating at any point during the day whilst ensuring that the demand for electricity is satisfied is known as the unit commitment problem. The power utility is faced with determining a 24-hour generation plan for each turbine given that the demand later in the day is not known with certainty. In this paper a method combining stochastic linear and stochastic dynamic programming is presented for determining a generation schedule for each turbine that hedges against the uncertainty in the future electricity demand.

1 Introduction
A hydro-electric power utility is typically situated along a river valley. The river valley consists of a number of reservoirs and a network of connecting waterways. When released, water stored in a reservoir flows through the river system into the next reservoir downstream. Hydro-electric power stations positioned between adjacent reservoirs house a number of generation turbines which operate in parallel. Water flowing through the station drives the turbine units which in turn generate electricity. The local manager of the hydro-electric scheme is faced with the problem of how to schedule the water releases and turbine units of the power utility. This must be done such that the required level of electricity generation is met whilst ensuring that water is available when required and not wasted unless it is completely unavoidable. This is known as the unit commitment problem.

The controlling distribution authority of a larger electricity generation and transmission system, of which a hydro-electric power utility is a part, coordinates the generation of electricity. Provisional 24-hour electricity requirements are issued by the distribution authority to each of the generation facilities. Unfortunately, the demand for electricity is inherently uncertain. Consequently, any variations in the provisional requirements has to
be accommodated in real time by the power utilities. This requires local managers to determine the optimal operation of their power scheme given that the demand for electricity over the day is not known.

2 Modelling the Uncertainty in Electricity Demand

Local managers strive to develop efficient water release and unit commitment schedules that are "well-hedged" against the uncertainty in the demand for electricity. Consequently, the times of most interest during the day are those when the decisions made have a significant impact upon the system. These times correspond to the periods in which the greatest uncertainty in the demand for electricity occur. That is, during the early morning low and the subsequent "ramping up" and peaking of the demand. It is at these times that economic decisions regarding the trade-off between the value of water and the cost of starting a new turbine must be made. For example, consider the case where the early morning low in electricity demand is less than expected. A decision must be made as to whether one of more turbine units should be shut down, or whether the current unit commitment should be maintained, but with a number of turbines generating less efficiently. If turbines are shut-down, these units will need to be started up again, over the next few periods, as the demand for electricity begins to increase. Thus, a fixed start-up cost will be incurred for each turbine. Conversely, should various turbines be kept operating, but less efficiently, the water flow through the station is not exploited to its fullest potential.

![Figure 1: Electricity demand scenarios for a hydro-electric power utility.](image)

To model the uncertainty in electricity demand, different possible electricity requirements over the day are considered. Each of the different requirements is known as a demand scenario and represents a limited amount of information about the demand uncertainty. The set of scenarios can be partitioned at each time period into a finite number of disjoint sets termed *scenario bundles*. At time \( t \), scenario \( \omega \) belongs to scenario bundle \( B_t^\omega \). Hence, if scenarios \( \omega \) and \( \omega' \) occur in the same bundle, \( B_t^\omega = B_t^{\omega'} \). Scenarios
in a particular scenario bundle at time $t$ can be thought of as having the same electricity requirements, that is, the same history, up until and including time $t$. To ensure that different scenarios from the same bundle do not anticipate their uncertain future electricity requirement, the decisions made under each of these scenarios, prior to time $t + 1$, must be the same. These decisions are said to be nonanticipative. In subsequent time periods, $B_t$, is refined into smaller disjoint bundles. At time $T$, each bundle contains a single unique scenario. Consequently, the scenario structure can be thought of as a tree. Each scenario corresponds to a path from the root of the tree out to a leaf. Figure 1 depicts some possible demand scenarios for a hydro-electric power utility and highlights the tree structure that results from scenario bundling. A scenario generation technique incorporating cluster analysis is being investigated to generate realistic demand scenarios from historical data.

3 Modelling a Hydro-electric Power Utility

Traditional optimisation based formulations of the deterministic unit commitment problem lead to large-scale mixed-integer nonlinear programming problems. Various decomposition techniques have been proposed to solve these computationally expensive problems [1, 2]. Until recently incorporating the natural uncertainty that exists in the problem into the model has been largely ignored in the literature. Takriti et. al. [3] incorporate demand uncertainty into a hydro/thermal unit commitment model which is solved using a scenario analysis technique. In this paper a method combining stochastic linear and stochastic dynamic programming is presented for determining a 24-hour hydro-electric generation schedule for each turbine that hedges against the uncertainty in the future electricity demand.

3.1 Scheduling the water releases

Synergies between turbines within the same power station prevent the treatment of each turbine in isolation. Similarly, there exits interdependencies between stations. Decisions made at one station have an impact upon other stations in the river valley. By assuming that the water flowing through a power station flows into a waterway and does not empty directly into the headpond of another station, the power stations can be treated independently. However, the time taken for water to travel along each waterway stills couples the system. Consequently, the determination of water release schedules and station unit commitments requires the consideration of the hydro-electric power utility as a whole and is non-trivial.

Given a particular unit commitment for the hydro-electric power utility a stochastic linear program can be solved to determine the schedule of water releases. The 24-hour time horizon is discretized into 30 minute intervals and the stochastic linear program is formulated as the following deterministic equivalent program.
In this formulation $x^\omega_t$ is the volume of water that flows along each waterway during time $t$ under scenario $\omega$ while $y^\omega_t$ denotes the water stored at each reservoir in the hydro-electric scheme at the end of time $t$ under scenario $\omega$. In the objective function (1), $p^\omega$ represents the probability with which scenario $\omega$ occurs and $v_i$ is the value of the water stored in each reservoir in the hydro-electric scheme at time $t$. Constraint (2) ensures that water is conserved throughout the system. The matrix $A$, is the node-arc incidence matrix of the hydro-electric scheme's time-expanded river network with arc delays and storage arcs (see Ford and Fulkerson [5]). The known vector, $r$ denotes the initial reservoir storages, the entering in-transit flows and the runoff from the catchment area into the respective reservoirs. In-transit flows are water releases along waterways with traversal times such that the water arrives at the next reservoir in a day following that in which it was released.

The generation constraint (3) ensures that the total electricity generated by the power stations satisfies the demand at all times. The electricity generated by each power station is dependent upon the flow of water through the station, the efficiency of the turbines operating within the station, and the headwater and tailwater elevations of the water flow above and below the station respectively. However, by stating certain assumptions, a station's generation curve can be modelled as a nonlinear concave function of the flow through the station [4]. The function $h^\omega_f(\cdot)$ determines the total generation of the power utility at time $t$ under scenario $\omega$ using piecewise linear approximations to each power station's nonlinear concave generation function. The demand for electricity at time $t$ under scenario $\omega$ is given by $g^\omega_f$.

Constraint (4) requires nonanticipativity upon of the water release variables. Constraint (5) imposes lower and upper bounds, $l$ and $u$, respectively, upon the waterway flows. The storage capacity of each reservoir is restricted to be $a$ in Constraint (6).

The schedule of water releases obtained from such a program is highly dependent upon the hydro-electric schemes initial conditions. If the reservoir storage levels from the previous day are low, then there may be insufficient water to generate enough electricity to satisfy the demand. One approach that overcomes this difficulty is to add constraints to the model that require the end conditions to equal the initial conditions. That is, storage levels of the reservoirs at the end of the horizon are the same as the initial reservoir storage levels and leaving in-transit flows are equal to the entering in-transit flows. Consequently, the hydro-electric scheme is left in the same state in which it began. As a result of these restrictions it is likely that the release schedule obtained will be quite conservative. An alternative approach is to relax the constraints requiring the
end conditions to be equal to the initial conditions and impose this requirement as a soft constraint using penalty functions.

3.2 Scheduling the unit commitment

The problem of determining a unit commitment has not yet been discussed. Craddock [4] describes an algorithm that operates in an iterative manner, determining the turbine allocation of each station in turn. The algorithm first determines an initial, possibly infeasible, schedule of water releases for each station from the deterministic equivalent program, using each station's composite generation function. These functions are defined as the concave hull of the station generation curves for the different possible unit commitments. For a particular unit commitment, a station's generation function is easily determined from empirical data. This schedule is known as the relaxed solution.

Secondly, the method seeks to minimise the cost of deviating from the relaxed operating levels when determining a feasible allocation of operating turbines for a particular station. Finally, with this station's allocation and previous station's allocations enforced, a new schedule of optimal water releases is determined.

To determine a unit commitment in the presence of uncertain demand, the stochastic nature of the problem must be incorporated into the algorithm. This suggests the use of a stochastic dynamic programming framework to determine the turbine allocations that minimise the cost of deviating from the relaxed operating levels found by the deterministic equivalent program. A stochastic dynamic program with backward recursion is applied to a state space determined by the electricity demand scenario tree (see Figure 1). Each state in the first stage, at time $T$, corresponds to a particular scenario bundle, $B_T$, each of which contains a unique scenario, $\omega$. When moving from one stage to the next, the state space of the stochastic dynamic program "collapses" according to the electricity generation requirements prescribed by the demand scenario tree. In the final stage, there exists a single scenario bundle representing the deterministic electricity demand requirement of the first time period.

The stochastic dynamic programming recursion is defined by

$$
F勃 (M) = \min_N \left\{ s(M,N) + d勃 (N) + \sum_{\omega \in B_t} p^\omega F勃_{T+1} (N) \right\}, \quad B_t \in B_t
$$

where $F勃_{T+1} (M) = s(M,N_{T+1})$ and $B_t$ is the collection of all scenario bundles at time $t$. At time $t$, under $\omega$, the expected cost of deviating from the relaxed operating levels in prescribing a unit commitment for the remainder of the time horizon is $F勃_t (M)$, where $M$ is the turbine allocation at time $t - 1$. The cost of switching from allocation $M$ to allocation $N$ is denoted by $s(M,N)$. At time $t$ under scenario $\omega \in B_t$, the cost of deviating from the relaxed operating levels to turbine allocation $N$ is given by $d勃_t (N)$.

An issue left largely unresolved by Craddock [4] was the order in which power stations were chosen in the turbine allocation algorithm. Various approaches are suggested but without any justification. One approach is that for each iteration of the algorithm the station with the largest $L_1$-norm of the expected marginal water values is selected from those stations still to be scheduled. That is, the station where it is most expensive to deviate from the relaxed operating levels has its unit commitment determined next. It
is envisaged that as the last few stations come to be scheduled, the electricity demand still to be accounted for will have an erratic profile. This demand will be best generated by stations with the smallest marginal water values. However, the effectiveness of this technique of station selection needs to be examined further.

4 Conclusions

A solution approach for determining water release and unit commitment schedules has been developed for a hydro-electric power utility where the demand for electricity is inherently uncertain. An alternative technique for generating demand scenarios has been mentioned but further investigation into the use of cluster analysis as a tool for scenarios generation is necessary before its effectiveness can be concluded. Continued development of the release scheduling model is suggested with the possibility of including penalty functions. Further investigation into the order in which the turbine allocation algorithm chooses stations to be scheduled is also needed.

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References


