TOWARDS ORTHOGONAL FLOORPLAN DESIGN

JOHN W. GIFFIN
MASSEY UNIVERSITY, PLAMERSTON NORTH, NZ

INGRID RINSMA
UNIVERSITY OF WAIKATO, HAMILTON, NZ

ABSTRACT
A method is described for producing an orthogonal geometrical dual satisfying the area and
adjacency prescription given by a vertex-weighted planar graph.

INTRODUCTION
An dimensioned orthogonal floorplan has a rectangular plan boundary and is divided into
rectangular rooms whose walls are parallel to the sides of the plan boundary. Each room \( M \)
has area \( a(M) \). L-rooms and T-rooms have the obvious shape definition. Two rooms are
adjacent if they have some positive length of wall in common. External rooms are adjacent to
the exterior region while internal rooms are not. To each floorplan there corresponds an
adjacency graph whose vertices represent the rooms and the exterior, and whose edges reflect
adjacencies. Deleting the exterior from the adjacency graph yields the so-called weak dual.

A graph is maximal planar if and only if it is planar and every face is a triangle. It is
sufficient to consider only maximal planar graphs in the context of floorplans. A graph is
outerplanar if it can be embedded in the plane so that all its vertices lie on the exterior face.
Further, a graph is maximal outerplanar if edge augmentation destroys outerplanarity.

BACKGROUND
Techniques in the area of graph-theoretic floorplan design are of current interest especially
because of large-scale integrated circuit technology requirements [6].

Earl [2] showed that many of the existing approaches are based on the same underlying
principle. Earl and March [3] derive the required conditions for a graph to represent the
adjacency structure of a floorplan whose rooms are all either rectangular or orthogonal. In a
rectangular case, a procedure based upon edge colouring of the graph enables a corresponding
floorplan to be derived in time proportional to the number of rooms [1].
However when room areas are included, existence of a rectangular floorplan cannot always be
guaranteed [7]. Further it is not always possible to have each room rectangular or L-shaped
[9], whereas for a general vertex-weighted maximal planar graph, it is possible under certain
conditions to achieve each room as rectangular, L-shaped or T-shaped [4]. For a summary of
the work in graph-theory based floorplan design since 1970, refer to Steadman [10].

In this paper we summarize [8] for outerplanar graphs and show how it may be extended to
construct an orthogonal floorplan for any vertex-weighted maximal planar graph. Practicalities
usually require that room shape be as regular as possible, but this is not our current objective.

PREPROCESSING THE ADJACENCY GRAPH

Given a maximal planar graph $G$, the vertices may be partitioned into sets $D_i$, $i \geq 1$, called
distance classes, where $D_i$ is the set of all vertices of (minimum) distance $i$ in $G$ from vertex
1 (the exterior). If $G_i$ is the subgraph of $G$ induced by $D_i$, then

(i) $G_i$ is outerplanar
(ii) Each block of $G_i$, $i \geq 2$, is either a single vertex, a bridge, or a 2-connected
outerplanar graph.

For any maximal outerplanar graph it is possible to impose a relabelling on its vertices so that
every vertex $i$, $i \geq 4$, is adjacent to two vertices $j$ and $k$ where $i > j > k$. Vertex $i$ is called
the successor of vertex $j$ and vertex $j$ the precursor of vertex $i$. Given any weighted maximal
outerplanar graph $G$ it is possible to construct a floorplan in which each room is rectangular
or L-shaped [8] satisfying the area and adjacency constraints of $G$.

For any edge $e = ij$ in $G$, the graph resulting from a contraction of $j$ to $i$, whereby vertex $j$
is removed and all edges previously incident upon $j$ are made incident upon $i$, with all
consequent loops deleted and multiple edges coalesced, is denoted by $G/ij$. A weighted
contraction also involves the augmentation of room areas.

ORTHOGONAL DIVISION ALGORITHM

Phase 1:
Identify $i = \arg \max_j \{D_j\}$.
Successively contract the vertices of each component of $G_i$ to a single vertex, noting the order
of contraction and the precursor and successor at each stage. These contracted vertices are
then further contracted to distinct vertices of $D_{i-1}$, resulting in a new maximal planar graph, $G'$. Continue this process until $G^*$ is obtained. Note that $G' = G'_{j'} - \{1\}$ is a maximal outerplanar graph.

**Phase 2:**
Using [8], as above, we construct a floorplan in which each room is rectangular or L-shaped, having $G'$ as its weak dual.

Iteratively, we reverse the contraction ordering imposed above, dividing a room, $X$, currently in the floorplan into two rooms, $X$ and $W$, where $W$ is the successor of $X$, maintaining orthogonality, (augmented) areas and (augmented) adjacencies. The essential idea is to develop an approximation of the shape of $W$ (relative to $X$) that meets the adjacency requirements of $W$. This approximation will not in general have the correct area, $a(W)$, so we successively perturb it incrementally whilst maintaining the adjacencies, orthogonality and contiguity (of $W$).

Let the required adjacencies of $W$ be $\{N_1, ..., N_s\}$ in order around the boundary of $W$. This is a subset of the neighbours of $X$. Within $X$, construct a rectilinear path (comprising a set of walls) with the minimal number of corners, drawn parallel to the boundary of $X$, between appropriately chosen points $\alpha$ and $\beta$ on the boundary of $X$ (on a wall of $N_1$ and $N_s$ respectively). $L$ divides $X$ into two rectanguloids, $P$ and $Q$, which are approximations to $X$ and $W$ in that they have the required adjacencies but arbitrary areas. For the case where $a(W) < a(Q)$, the perturbation consists of dividing $W$ into rectangles strictly interior to $X$. These are successively augmented to $P$, ensuring that each one added is adjacent to $P$, until $a(Q) = a(W)$. In the other case ($a(W) > a(Q)$) a similar process is used, except that rectangles are added to $Q$ rather than $P$.

Experience thus far indicates that non-pathological adjacency graphs yield floorplans with reasonable roomshape. We emphasize that our approach can dualize any adjacency graph, in contrast with the classic maximal planar boustrophedon approach [5] which cannot handle the existence of a separating triangle in an adjacency graph. A disadvantage is that computer implementation of our method will not be so readily attainable. However, we note that implementation should be possible in time linear in the number of vertices of $G$, since each phase of the algorithm depends only upon the unique precursor-successor relationship, provided that an upper bound on the number of "unit squares" required by the perturbation phase is determinable.
REFERENCES


