ECONOMIC ORDER QUANTITIES FOR A FINITE DEMAND HORIZON WITH A SINGLE PRICE CHANGE - A NOTE

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SUMMARY

An inventory model involving a price change over a finite horizon is investigated. The model assumes a one-time increase at some intermediary point in the horizon. An example is provided to illustrate the use of the model.

1. INTRODUCTION

In the classical EOQ formula, one of the basic assumptions is that the per unit price of the product will remain the same over the planning horizon of interest. Unfortunately, this is not always a realistic assumption. Frequently there are expected increases in the unit price of the product during the course of the planning horizon. This may be a result of inflation or a variety of other causes.

In recent years a number of models have been proposed for dealing with such changes [1-4,6,8]. In particular, Naddor [8] considers a situation in which a one time price increase is to occur at a known time S. It is assumed that the inventory level is zero at S and the option is available to purchase a larger amount at S just before the price change occurs. Lev and Soyster [6] extend Naddor's model to the case in which the inventory level need not be zero at time S. Both models assume that the planning horizon is infinite. The purpose of this paper is to consider a model pertaining to a one-time increase in the purchase price over a finite horizon. For many situations this is a more realistic assumption since the mere recognition that the purchase price is changing implies that the new purchase price will only be in effect for a limited time before another change occurs.
2. A ONE-TIME INCREASE IN PURCHASE PRICE WITH A FINITE HORIZON

For this model the following assumptions will be made:
(1) $D$ is the yearly demand for the product; (2) the demand rate is uniform; (3) shortages are not allowed; (4) the lead time for a purchase order is zero; (5) $k$ is the order cost per order; (6) the purchase price is initially $C_1$ per unit and then changes to $C_2 > C_1$ per unit at some known time $S$; (7) the interval $[0, T+S]$ is the finite horizon of interest; (8) the holding cost is proportional to the unit price with $r$ being the proportionality constant. Assumption (7) is different from both Naddor's and Lev and Soyster's models (despite the title) in that a finite value is assumed for $T$.

It has been demonstrated that the optimal policy over an infinite horizon when the purchase price changes from $C_1$ to $C_2 > C_1$ at time $S$ consists of either
(a) placing $n^*$ orders equal to $DS/n^*$ followed by one order of size $Q_b$ and then each order equal to $Q_\infty$, or
(b) placing $n'$ orders each equal to $Q_b$ and then each order equal to $Q_\infty$,

where

\[
n^* = \langle -\frac{1}{2} + \sqrt{DrC_1S^2/2k + \frac{1}{4}} \rangle
\]
\[
Q_b = \frac{D(C_2-C_1) + \sqrt{2rkDC_2}}{rC_1}
\]
\[
Q_\infty = \sqrt{2kD/rC_2}
\]
\[
n' = \langle DS/Q_b \rangle
\]

and $\langle x \rangle$ is the smallest integer greater than or equal to $x$ [5,7].

Policy (a) has zero inventory at $S$, while policy (b) does not have zero inventory at $S$, except by coincidence. Neither of these policies, however, is necessarily optimal if the inventory level is constrained to be zero at time $T+S$.

In order to adjust these policies to make them optimal in the finite horizon $[0, T+S]$, the following adjustments must be made:

(a') place $n^*$ orders each equal to $DS/n^*$ followed by one order of size $R$ followed by an optimal ordering policy on the interval $[S+R/D, T+S]$, (b') place $n'$ orders each equal to $R$ followed by an optimal ordering policy on the interval $[n'R/D, T+S]$. 
The optimal ordering policy referred to in (a') consists of placing \( n_1 \) orders of size \( Q_1 = (DT-R)/n_1 \) so as to minimize the total cost on \([S+R/D,T+S]\)

\[
K(n_1) = n_1k + C_2(DT-R) + (DT-R)^2rC_2/2Dn_1
\]

The optimal value of \( n_1 \) is \( n_1 \) such that \( K(n_1) \leq K(n_1+1) \) and \( K(n_1) \leq K(n_1-1) \). It follows that

\[
n_1 = \left( -\frac{1}{2} + \sqrt{DrC_2(DT-R)^2/2k + \frac{1}{4}} \right)
\]

Similarly, the optimal order policy referred to in (b') consists of placing \( n_2 \) orders of size \( Q_2 = (T+S-n'R/D)/n_2 \) so as to minimize the total cost on \([n'R/D,T+S]\),

\[
K(n_2) = n_2k + C_2(D(T+S) - n'R) + (T+S-n'R/D)^2rC_2/2Dn_2
\]

In this case the optimal value of \( n_2 \) is

\[
n_2 = \left( -\frac{1}{2} + \sqrt{DrC_2(T+S-n'R/D)^2/2k + \frac{1}{4}} \right)
\]

These rules are similar to the rule presented by Schwartz [9] for determining the optimal number of orders in finite horizon problems without price changes. Also, the method of determining (2) and (4) from (1) and (3), respectively, is identical to that presented by Lev, Soyster, and Weiss [7].

3. AN EXAMPLE

As an example of this type of problem, suppose the following data are available for a given product: \( C_1 = $5, C_2 = $6, k = $50, r = 0.10, D = 10, S = 24, T+S = 44 \). This is similar to a problem provided by Lev and Soyster [6], except for the specification of the finite time horizon of 44. If the infinite horizon models of Lev and Soyster are used to solve this problem, policy (a) consists of ordering 69 units, followed by three orders of size 40.82 and one order of 8.54. The total cost of this policy is $3390.18. Policy (b) of Lev and Soyster consists of 4 orders of 69 followed by 4 orders of 40.82 and one order of 0.72. The total cost is $3013.97. The finite horizon model of policy (a') consists of ordering 69 units followed by three orders of 43.67. The total cost of this policy is $3359.66. Policy (b') consists of four orders of 69 followed by four orders of 41 with a total cost of $2965.72. Thus policy (a') is superior to policy (a), and policy (b') is superior to policy (b), and the best policy for this finite horizon problem is (b').
REFERENCES


