INVESTMENT APPRAISAL UNDER UNCERTAINTY: 
ALLOWING FOR CORRELATIONS IN PROJECT VARIABLES*

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SUMMARY

A review of procedures for assessing the profitability of proposed investments under uncertainty suggests that inadequate attention has been paid to correlations in the variables which determine project cash flows. A model for evaluating investments characterised by such correlations where cash flow variables can be represented by a conditional multivariate normal distribution is presented. This model is applied to a proposed real estate development venture for which uncertain revenue and cost items are considered to be both correlated with each other and autocorrelated over time. Simulated results for the venture, expressed as probability distributions of discounted cash flow profitability criteria, reveal the impact of allowance for correlations on estimates of investment risk.

1. INTRODUCTION

Management of business firms frequently have cause to evaluate investment opportunities, the costs and returns of which extend a number of years into the future. Uncertainty clearly characterises these outcomes. Traditionally, various deterministic project evaluation models have been employed which involve the calculation of static criteria such as annual percent return on capital and pay-back period or dynamic discounted cash flow criteria. Uncertainty is taken into account in these models, if at all, by crude procedures such as conservative cash flow estimates, risk premia incorporated in discount rates or sensitivity analysis of performance with respect to some 'critical' project variables. The advent of "risk analysis" (Hertz [8]; Klausner [11]; Cassidy et al. [2]; Pouliquen [17]; Hayes [7]; Hull [9]) and the development of analytical project evaluation methods (Hertz [8]; Wagle [20]; Reisman and Rao [18]; Zinn et al. [20])

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have provided superior approaches in that an attempt is made to quantify uncertainty in project input variables through estimation of parameters of their subjective probability distributions. In a risk analysis, Monte-Carlo sampling is then carried out from these input distributions, cash flows are calculated and one or more profitability criteria are derived. Repetition of this procedure (i.e. replication) allows project profitability to be expressed in distribution form, e.g. results may be depicted as a graph of cumulative "less than" relative frequencies of say, net present value. Analytical models, on the other hand, allow derivation of closed-form precise expressions for the expected value and variance of profitability criteria under a restricted range of relationships between project variables.

Risk analysis has had very wide acceptance among management scientists [7] and appears to be the most favoured method of evaluating investment risk in industry [20]. However, it is our contention that in spite of its wide use, the technique frequently is used without proper recognition of its inadequacies. The failure to take full account of correlations between and within project variables such as product price, input costs and project life can be documented in a range of published accounts using risk analysis techniques. Given such correlations are overlooked, or treated inadequately, an inaccurate measure of performance variability may be obtained. For instance, suppose various cost items are positively correlated with each other. In this case, independent sampling may lead to inappropriate cancelling of extreme values in the different items and may thus underestimate cost variability. Again, suppose that product price has high positive autocorrelation over time; that is, runs of a number of years above and below the long-term trend take place. Runs of high prices may result in a high level of profitability for the proposed venture; runs of low prices may be disastrous. Independent sampling for successive time periods, as is the usual approach, may thus
result in runs being under-represented, and cause project performance variability to be under-estimated. For risk-averse management and a marginally profitable project, inclusion of correlations could well reverse the decision to accept the proposal.

The central theme of this paper is to illustrate, within the general framework of risk analysis, how correlations in project variables can be allowed for and how sensitivity to alternative assumptions regarding these issues can be explored. This exposition progresses by firstly reviewing procedures (however inadequate) which have been employed in past studies for dealing with this problem. Then the sampling method which has been devised in this study is explicated. Next, a case study dealing with real estate investment is described, and the accounting model for determining projected profitability estimates of the venture is presented. The final section examines the effect of different types and levels of correlations on the distributions of profitability criteria, both for the actual case study employed and in general terms.

2. A REVIEW OF PROCEDURES FOR INCORPORATION OF CORRELATION

While early risk analysis models tended to overlook correlations in project variables, some subsequent studies have, in part measure, considered this problem. In an early example, Pouliquen [17] expressed the view that

"Correlations are very difficult to detect, and even more difficult to measure, but overlooking them may lead to a completely wrong interpretation of the analysis". (p.45).

This World Bank researcher went on to suggest aggregation of variables (i.e., use of unit correlations) to measure the limiting effect of the phenomena. One disadvantage of this suggestion might be noted; it does not show how less than perfect correlation may be included in the analysis. A more flexible procedure, when dealing with two variables, is to draw up conditional distributions of one for each of a
number of discrete values of the other (Beenhakker [1]; Hull [9]; Van Horne [19]) with sampling being carried out sequentially on the two variables. Both approaches outlined are rather inflexible. Certainly they appear less satisfactory, by comparison, with the adoption of a single multivariate distribution. Sampling from the multivariate normal distribution, while now well established in simulation methodology, unfortunately does not appear to have received attention concerning risk analysis of proposed investments. Perhaps one reason why this neglect has arisen is that the marginal distribution of each variable is restricted to a normal probability function. Project appraisal experience by the International Bank for Reconstruction and Development, among others, suggests that the normal curve does not accord with subjective input variable distributions elicited from experts [17]. However, the multivariate normal distribution is particularly amenable for sampling purposes, and errors from failure to include correlations between variables may be much greater than those due to lack of fit of the normal curve. In any case, by use of appropriate transformations even highly skewed distributions may be made to approximate the normal curve, as demonstrated by Harrison [6].

Progress has been made in addition, towards an inclusion in risk analysis of autocorrelation within a particular variable over time. When the variable is normally distributed, a procedure derived by Llewellyn [12] allows consideration of any level of autocorrelation: this procedure has been applied to product price in the evaluation of an agricultural investment by Dent [3]. However, simultaneous inclusion of correlation between, and autocorrelation within, project variables presents a more difficult statistical problem. For this purpose, Johnson [10] has combined sampling from a bivariate normal distribution with exponential smoothing. In this paper we present a more general and versatile approach which can be used for any number of interrelated variables, each of which may have a different level
of autocorrelation. The procedure is applied herein to examine explicitly the effects of both forms of correlations in an investment setting on the distributions of net present value and peak deficit, using as a vehicle of exposition a postulated real estate venture. The case study utilizes an accounting model which is intentionally simple, and with minor modifications, would have been amenable to an analytical evaluation method, at least with regard to the derivation of net present value. However, the procedure advanced is regarded as being more robust than closed-form alternatives and capable of dealing with situations where complex relationships exist between project variables. Before proceeding to the project example, it is helpful to give a statistical description of the sampling method incorporated in our methodology.

3. THE SAMPLING PROCEDURE

Suppose the "state of nature" in any time period $t$ may be represented by levels of $p$ random variates $Y = (y_1, y_2, \ldots, y_p)$. Further suppose these variables are interdependent and exhibit first order serial correlation, i.e., $Y_i, t = f(y_1, t, \ldots, y_{i-1}, t, y_{i+1}, t, \ldots, y_p, t, y_{1, t-1}, \ldots, y_{p, t-1})$. Now let $y_{i, t-1} = y_{p+i}$, for $i = 1, \ldots, p$. The vector of random variables is then augmented to $Y = (y_1, y_2, \ldots, y_{2p})$ which follows a multivariate relationship with mean vector $\mu = [\mu_1, \mu_2, \ldots, \mu_{2p}]$ and covariance matrix

$$V = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1,2p} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{2,2p} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{2p,1} & \sigma_{2p,2} & \cdots & \sigma_{2p,2p}
\end{bmatrix}$$

If the time series for each variable is stationary and homoscedastic (i.e., has constant error variance), then

$$\mu_{p+i} = \mu_i, \quad i = 1, \ldots, p$$

and

$$\sigma_{p+i, p+j} = \sigma_{ij}, \quad i = 1, \ldots, p, \quad j = 1, \ldots, p.$$
Recalling that the coefficient of correlation between variables is given by \( \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \) where \( \sigma_i = \sigma_i^{1/2} \), it may be noted that for matrix \( V \)

\[
\rho_{i,p+i} = \frac{\sigma_{i,p+i}}{\sigma_i \sigma_{p+i}} = \sigma_{i,p+i}^{1/2}
\]

expresses the first order autocorrelation coefficient for variable \( i \).

The covariance matrix may be partitioned as

\[
V = \begin{bmatrix}
V_{11} & V_{12} \\
V_{21} & V_{22}
\end{bmatrix}
\]

where \( V_{22} = V_{11} \) and \( V_{21} = V_{12} \). Similarly, the vector of means may be partitioned into \( \mu = [\mu_1, \mu_2] \) where \( \mu_2 = \mu_1 \). If the vector \( Y \) follows a 2p-variate normal distribution, i.e.,

\[
f(Y) = f(y_1, y_2, \ldots, y_{2p}) = \frac{1}{(2\pi)^{p} |V|^{1/2}} e^{-\frac{1}{2}(Y-\mu)'V^{-1}(Y-\mu)}
\]

then \( y_1, \ldots, y_p \) given \( y_{p+1}, \ldots, y_{2p} \) (i.e., the levels of the \( p \) variates in period \( t \) given their levels in \( t-1 \)) follow a \( p \)-variate conditional normal distribution. More specifically, if we let \( Y = [Y_1 Y_2] \), where \( Y_1 = (y_1, y_2, \ldots, y_p) \),

\[
Y_2 = (y_{p+1}, y_{p+2}, \ldots, y_{2p})
\]

and asterisks are used to indicate particular values of the variables, then \( h(Y_1 | Y_2 = Y_2^* \) follows a \( p \)-variate conditional normal distribution with mean vector

\[
\mu_*^{*} = \mu_1 + V_{12} V_{22}^{-1} (Y_2^* - \mu_2)
\]

and covariance matrix \( \{13, \text{p.213}\} \)

\[
\Sigma_*^{*} = V_{11} - V_{12} V_{22}^{-1} V_{21}
\]

Noting that \( \mu_2 = \mu_1 \), these means and variances may alternatively be written as

\[
\mu_*^{*} = D \mu_1 + E Y_2^* \quad \text{and} \quad \Sigma_*^{*} = V_{11} - E V_{21}
\]

where \( E = V_{12} V_{22}^{-1} \), and \( D = I - E \). Sampling from the distribution \( h(Y_1 | Y_2^*) \) utilizes the relationship

\[
Y_1^* = CZ + U_1^*
\]

where \( Z \) is a vector of independent standard normal variates and \( C \) is the unique lower triangular matrix such that
CC' = \( V^{*}_{11} \). The elements of C may be obtained by the "square root method" [14] or "Crout Factorization" of \( V^{*}_{11} \) [15].

4. THE INVESTMENT PROJECT AND ACCOUNTING MODEL

As indicated previously, a real estate investment proposal is analysed as a vehicle for exposition of the methodological innovations espoused. The real estate market is recognized as one of high risk and one in which the form of investment appraisal traditionally takes inadequate cognizance of uncertainty in profitability projections [16]. The real-world case study analysed here involved the decision to purchase or not to purchase a discrete parcel of fringing near-city acreage in the Moreton Region, south-east Queensland, for subdivision and sale as residential and industrial allotments. The land has been surveyed and local authority approval has been obtained for a design which provides 443 equivalent building blocks, as part of the concept of fostering and developing a new town in the region. For our purpose, multiple dwelling residential lots, industrial and business areas and schools, are expressed herein on a residential block equivalent basis. Blocks are to be released over a four-year period. Budgets have been prepared of current expenditure levels, including initial purchase of land and development costs (this latter item encompassing roadworks, water supply, sewerage and local authority contributions for headworks.)

The risk analysis model generates sequences of project variables by pseudo-random sampling from estimated subjective normal probability density functions of critical inputs and outputs. For each simulated sequence, cash flows are budgeted and discounted cash flow profitability measures are derived. Finally, these measures are expressed in cumulative relative frequency form and parameters of their distributions are estimated. For the present exposition, performance criteria have been confined to net present value (NPV) and peak deficit (PD), defined respectively as
\[ \text{NPV} = \sum_{t=0}^{n} CF_t \left(1 + \frac{i}{100}\right)^{-t} \]

and

\[ \text{PD} = \min \sum_{j=0}^{n} CF_j \left(1 + \frac{i}{100}\right)^t, \quad j = 0, 1, \ldots, n, \quad t = 0 \]

where \( i \) is the discount rate, \( n \) is project life, and \( CF_t \) is the net cash flow in year \( t \). The model assumes that \( i \), the weighted average cost of capital from various sources, is constant over time and known with certainty.

Components of cash flows include annual revenues and costs which are defined as follows:

\[ F_t = B_t E_t, \]
\[ G_t = B_t A_t + C_t + D_t + Q_t + T_t, \]

where \( B_t \) is aggregate block output, and \( E_t \) is price per unit (block), \( A_t \) denotes per unit costs associated with output, and the remaining cost items refer respectively to wages, other operating expenses, plant leasing costs and taxation.

To relate these items in terms of the particular model to describe actual project cash flows, the following definition is applicable:

\[ \begin{cases} 
- P - W - \ell_1 G_{t+1}, & t = 0 \\
\ell_2 F_t + (1 - \ell_2) F_{t-1} - \ell_1 G_{t+1} - (1 - \ell_1) G_t, & 1 < t < n \\
F_t + (1 - \ell_2) F_{t-1} - (1 - \ell_1) G_t + S_t, & t = n
\end{cases} \]

Hence \( P \) is initial project outlay (the purchase of the land), \( W \) represents contingency expenses and \( S_t \) is the salvage value of the project, viz. zero in this study. Now \( \ell_1 \) and \( \ell_2 \) are the respective proportions of total annual costs \( (G_t) \), and gross annual income \( (F_t) \), led and lagged in time; both are set at 0.25 in the case study analysis. This feature of leading some expenses and lagging receipt of proportion of project income is considered as introducing realism, given the contracting and marketing conditions for land development expenses, and real estate term sales, respectively.
4.1 STOCHASTIC VARIABLES AND THEIR DISTRIBUTION

Major sources of financial uncertainty affecting project profitability are identified as the average sale price of allotments, land development costs and marketing expenses. It is anticipated that block sale prices \( (E_t) \) and development costs \( (L_t) \) will follow a rising trend of 10 percent per annum from current levels of $8000 and $4600, respectively. It is acknowledged that there may be wide fluctuations about this trend. Marketing expenses \( (M_t) \), as defined, consist of agents' commission \( (COMM) \) plus discretionary selling expenses, mainly for advertising. Algebraically these expressions are stated as:

\[
E_t = 8000 + 800 \, t + y_1
\]
\[
L_t = 4600 + 460 \, t + y_2
\]
\[
M_t = COMM + y_3
\]

and

\[
COMM = \begin{cases} 
0.05E_t; & E_t \leq 8000 \\
400 + 0.025 \, (E_t - 8000), & E_t > 8000 
\end{cases}
\]

The distributions of the variables \( y_i \) \( (i=1,3) \) are now considered in detail.

4.2 ESTIMATION OF DISTRIBUTIONS OF THE UNCERTAIN VARIABLES

Estimation of parameters of the distributions of uncertain project variates in risk analysis normally takes the form of elicitation of management's subjective beliefs about the future values of these variables. If their joint distribution is considered (rather than making the simplifying assumption that the variables are independent), then the task of elicitation may become quite difficult.

The conditional multivariate normal distribution, as introduced in Section 3, is specified in terms of a vector of means and a covariance matrix. For the real estate venture the variables \( y_1 \) and \( y_2 \) are deviations about time series trends and have zero means, while extra selling costs
\( y_3 \) are assumed to have a mean of $200. Estimation of the variances and covariances was more difficult. To obtain the former, two standard deviation ranges about the means were sought and these were estimated by management as $2400, $1000 and $400 respectively; corresponding variances are indicated on the principal diagonal of the covariance matrix (Table 1). Only the upper half of this matrix is provided since the lower half is defined by symmetry conditions indicated above. Covariance elements have been obtained indirectly from correlation coefficients, which were also elicited from management on a subjective basis. These estimates which were made by a member of the management team with formal training and experience in statistical methods, are not regarded as precise. Rather, the approach adopted has been to apply sensitivity testing to correlation levels. In this regard selling price and land development costs were considered to have a positive correlation of 0.5. Additionally, it was considered that the firm would increase marketing expenditure if selling cost is low in order to maintain the timing of sales; a correlation coefficient of -0.5 was estimated for these variables. Autocorrelation coefficients were specified as 0.5 for each of the three variables.

Other relationships between stochastic variables were considered to be unimportant; i.e., the partial correlation coefficients after allowance for the correlations specified above were taken as zero. It is to be noted here that the assumption that remaining total correlation coefficients are zero would be inappropriate. For three variables, if the first is correlated with the second and the second with the third, then upper and lower bounds are placed on the correlation between the first and third, resulting in a range which may exclude zero. Failure to observe these bounds may produce a correlation matrix which is not positive definite, and factorization of the conditional covariance matrix becomes impossible. Actual values of the bounds are difficult to determine, particularly when more than three variables
<table>
<thead>
<tr>
<th>$Y_6$</th>
<th>$Y_5$</th>
<th>$Y_4$</th>
<th>$Y_3$</th>
<th>$Y_2$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>3125</td>
<td>15000</td>
<td>10000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3125</td>
<td>0</td>
<td>31250</td>
<td>37500</td>
<td>6250</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0</td>
<td>37500</td>
<td>18000</td>
<td>6250</td>
<td>7500</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
Y_1 & Y_2 & Y_3 \\
0 & 0 & 0 \\
Y_2 & Y_4 & 0 \\
Y_3 & Y_1 & 0
\end{bmatrix}
\]

**Mean ($\mu$)**

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix}
\]

Normal Distribution of Stochastic Variables

Permutation of Conditional Multivariate

**TABLE**
are present, but are related to the mathematical requirements for a positive definite covariance matrix.

The partial correlation between \( y_i \) and \( y_k \) given the correlations between \( y_i \) and \( y_j \) and between \( y_j \) and \( y_k \) is defined as

\[
\rho_{ik,j} = \frac{\rho_{ik} - \rho_{ij}\rho_{jk}}{\sqrt{(1-\rho_{ij}^2)(1-\rho_{jk}^2)}}
\]

Thus if \( \rho_{ik,j} \) is zero the total correlation between \( y_i \) and \( y_k \) is given as \( \rho_{ik} = \rho_{ij}\rho_{jk} \). Having defined the variances and correlation matrix the covariances may now be estimated from the relationship \( \sigma_{ij} = \rho_{ij}\sigma_i\sigma_j \). For example, the element \( \sigma_{15} \) is obtained as

\[
\sigma_{15} = \rho_{15}\sigma_1\sigma_5 = \rho_{12}\rho_{25}\sigma_1\sigma_2 = 37,500
\]

The elements of Table 1 provide a basis for generation of the random components in the accounting model, matrix operations being performed on \( U \) and \( V \) to obtain the required sampling expression for \( Y_1 \). The procedure has also been repeated for certain variations of the covariance matrix which are now outlined.

4.3 THE RISK ANALYSIS

The proposed real estate investment has been subjected to risk analysis in which the stochastic variables have the vector of means and covariance matrix as indicated in Table 1. To examine the effect of errors in estimation of correlations (i.e. to apply sensitivity testing), the analysis has also been performed with all total correlation coefficients set at half of their above values. Four cases of correlation are examined for both levels of coefficients, viz.

1. all variables independent, i.e., \( \sigma_{ii} > 0 \), \( i = 1 \) to 6 and \( \sigma_{ij} = 0 \) elsewhere;
2. variables correlated but no autocorrelation, i.e., \( \sigma_{ij} \neq 0 \), \( i,j < 3 \) and \( i,j > 4 \), and \( \sigma_{ij} \neq 0 \) elsewhere;
3. variables autocorrelated but independent of each other,
4. variables correlated and autocorrelated, i.e. \( \sigma_{ij} \neq 0 \), all \( i, j \);

and

5. variables correlated and autocorrelated, and initial levels of random variables (in \( Y_t^\alpha \)) set at one standard deviation in the direction of reduced profitability. (Initial \( Y_t^\alpha = 0 \) for cases 1 to 4).

Case 1 provides a standard against which to compare the distributions of performance criteria when various forms and levels of correlations are included. Cases 2 and 3 decompose the effect of correlation according to its source, viz., between and within variables respectively, while the overall effect of correlation is represented by case 4. The introduction of adverse initial values in case 5 typifies the present economic climate of depressed land prices and associated high advertising costs; because of the positive autocorrelation the model tends to perpetuate current adverse values into future time periods. One hundred replications (i.e. 100 generated sequences of variables, estimation of cash flows and D.C.F. analyses) have been performed for each of the five cases.

5. RESULTS

Cumulative frequency curves for net present value and peak deficit are presented as Figures 1 and 3 (high correlation coefficients) and Figures 2 and 4 (low coefficients). An interest (discount) rate of 16 percent is used throughout. Figure 1 indicates that, relative to independent sampling, inclusion of correlation between variables moves the NPV ogive to the right at the bottom and to the left at the top, i.e., the project becomes less risky. This is because the two major sources of uncertainty, deviations about the price and development cost trends, act in opposite
directions with respect to NPV. These deviations are positively correlated so high (and low) values of each tend to occur in the same year with the result that the margin between price and cost is lower than had they varied independently. Inclusion of autocorrelation (case 3) has the opposite effect; frequencies of runs of the three variables above and below their trends (e.g., runs of low land prices) are increased, leading to a more variable NPV. A comparison of the desirability of different performance distributions on the basis of risk levels without restrictive assumptions about the managerial utility function (other than risk aversion) may be made using the concept of stochastic dominance [5,21]. Discussion here, however, has been limited to relative risk levels as indicated by the shapes of the NPV and PD curves.

The combined effect of correlation and autocorrelation is revealed in Table 1 as a small increase in the level of risk. Nowhere do the ogives representing the distribution of outcomes for the performance criteria depart very greatly for these four cases. At the zero NPV or break-even point cumulative frequency ranges from .13 (correlation only) to .20 (autocorrelation only). The inclusion of adverse initial values of the three variables (case 5) has a much greater impact. Not only is the shape of the NPV ogive changed, but the whole curve is shifted to the left, i.e., profitability is greatly reduced, the likelihood of a negative NPV now exceeding 0.4. This is perhaps the most realistic of the five cases; recent experience in the real estate industry bears out the possibility of stagflationary business cycle conditions with uncertain project variables remaining more adverse than their anticipated long-run trends. Peak deficit distributions in Figures 3 and 4 are limited to cases 1, 4 and 5 for simplicity of presentation. These tables reveal that introduction of correlations and autocorrelations makes the PD curves more vertical, i.e., variability of the deficit is reduced. Adverse initial values shift the curve to the
FIGURE I
CUMULATIVE NET PRESENT VALUE DISTRIBUTIONS (HIGH CORRELATION)
FIGURE 3
CUMMULATIVE PEAK DEFICIT DISTRIBUTIONS (HIGH CORRELATION)
FIGURE 4
CUMULATIVE PEAK DEFICIT DISTRIBUTIONS (LOW CORRELATION)

PEAK DEFICIT ($M$)

CUMULATIVE RELATIVE FREQUENCY

all variables independent correlation and autocorrelation correlation and adverse initial levels
right, implying an increase in the deficit at each probability level.

The effect of lower correlations and hence covariances is to narrow the band of curves and reduce the range of relative frequency for a given NPV or PD level. The general relationships between the curves as outlined above is maintained.

6. DISCUSSION AND CONCLUSIONS

The inclusion of correlations between and within project variables has changed the shape of the ogives for performance criteria derived for a proposed real estate venture used as a vehicle for exposition. Inclusion of correlation between variables alone has reduced the variability of net present value, due to compensating effects on income and cost items. This would probably be the case in many investments where levels of a range of financial variables move in sympathy with general economic conditions over time. However, cases could arise also where risk is heightened by correlations between variables. The effect of autocorrelation has been to increase variability through increasing the frequency of runs of values of input variables above and below their means. This would be the case in the majority of investments. In general, then, the two forms of correlations can be expected to work in opposite directions, and the net effect may be difficult to predict in advance. However, inclusion of correlations between variables alone may lead to an adjustment for risk in the wrong direction. This is the case for the real estate proposal where the net effect of the two forms of correlations is to increase variability of NPV. Thus consideration of correlations between variables alone, as has been advocated in investment analysis literature, may be highly misleading.

The procedure we have outlined should have wide applicability for risk analysis, although two important limitations are noted. The first is the assumption of normality. This,
as indicated earlier, may be overcome partially by the use of normalizing transformations in cases that demand them. The second limitation concerns elicitation from management of the covariance matrix. Although this problem arises in any form of risk analysis, more realistic modelling of uncertainty necessitates more sophisticated questioning procedures, and further research into techniques for estimating parameters of subjective joint distributions of uncertain project variables appears warranted.

REFERENCES


