SUMMARY

There have been many computerized schedulers that have attempted to solve the sheeted paper trimming problem. Most of these are lacking in some facet, and have proven difficult to implement.

Several of these schedulers and their methods are reviewed, along with the development of a model, which it is hoped will satisfactorily solve the sheeted paper trim problem, and may be extended to the reel problem.
i. **Introduction**

As paper machines represent enormous capital investments and are most expensive to run, it is of vital importance that they be utilised to their utmost capacity.

One of the ways of improving efficiency is to maximise their usable width. This is generally known as "The Paper Trim Problem".

A paper manufacturer may sell paper in the form of sheets or reels and thus the problem is often divided into "Reeled and Sheeted trimming". As a consequence this presentation includes the trimming of reels and sheets.

The production set up, as shown in Diagram 1, is as follows:

Paper is formed as a large reel at the end of the paper machine, and as this occurs a small amount of waste paper is trimmed off the edges. This waste is called safety trim and exists so that the reels have even and firm edges. These large reels are then passed to the winder.

At the winder, reels of smaller width are formed. Reel orders are satisfied at this stage. Hence trimming is first encountered at the winder.

If orders are sheeted, the reels are sent to a cutter. A cutter initially splices a reel that is unwinding, into smaller widths. These widths are then passed under chopping knives which produce the final rectangles.

For sheet production the winder is necessary only if the cutter has a smaller width than the paper.
machine.

A cutter may be simplex or a duplex. A simplex cutter can cut a set of widths into one length only, whereas a duplex can cut up to two lengths. A duplex cutter can be run as a simplex if desired, and when this is done production can be slightly increased.

ii. **The Scheduling Problem**

When preparing a production run the aim is to combine the orders in such a fashion as to minimise the loss of production. That is to utilise the paper machines full width, or something approaching it.

To help the manufacturer to do this the following aids are available.

1. **Over and Under Making**

   Within a fairly narrow range, it is possible to over or undermake an order, and thereby reduce the waste. For an example see diagram 2.

   The waste LMNO would be reduced to L'M'N'O' if order A was over made by 10% and order B undermade by 20%.

2. **Sheet Rotation**

   The fibres in paper tend to run parallel with the direction of travel in the machine. The fibre orientation is called "grain direction". Usually the grain direction of a sheet is important to a customer since it either enhances the sheets' appearance or adds to its strength. However, if a customer does not require these properties then the sheet may be
rotated to give a better cutting pattern.

3. **Stock Cutting**

The waste of a poor pattern may be stored for later use. This may be especially economical for small production runs that could be completely satisfied from inventory.

The problem so far seems aimed at minimizing the waste, but this should really be altered to minimizing costs. Consequently the following points must also be considered.

4. **Cutter Utilisation**

Consider a cutter of 70 inches and a paper machine of 120 inches, then a pattern using more than 70 inches of the paper machine requires at least two passes over the cutter. However, a pattern involving three passes e.g. the pattern 3 of 40" would be rejected as it would reduce production excessively.

5. **Costs**

To minimise cost each pattern must be debited for:

a. Waste. If there is W inches waste and the cost per inch per ton to run the paper machine on this grade is C dollars, then the waste cost becomes CW.

b. Cutter use. Let the costs of a simplex cutter be Cs dollars per ton and Cd for a duplex.
Note: that Cs is less than Cd. Hence the total cutter cost becomes $\sum Cs,d$.

e.g. the pattern 2 of 30" by 15" + 2 of 27" by 22" is charged with 2Cs not 2Cd.

c. Minimum run length. A desirable pattern should be of sufficient length so as to make it profitable i.e. There exists a break even point between profits and setup costs.

The number of patterns that satisfy all the constraints is phenomenal and selection of the minimum cost schedule becomes impossible manually with any regularity, although a skilled manual scheduler can often approach it. Despite the generally good manual schedules a computer program can still offer worthwhile gains.

iii. Algorithms for Computerised Schedulers

There seems to be two approaches to developing trim minimisation methods for use with computers.

1. Heuristic Methods

These simulate the actions of the human scheduler and thus the flexible decisions and control over patterns is inherent.

When the number of orders become small, they often cannot improve upon the manual efforts. Against this they are quite fast.

Very briefly these schedulers work as shown in Diagram 3.
DIAGRAM 3.

Select Key Order →

Pair with a Companion →

Update Stock

None

Is there a pattern for this key order?

Y

Schedule key order
By itself

N

Does pattern satisfy Min. Trim?

Match over/under makes

Compute Costs

N

Does Pattern satisfy Min. Cost constraint?

Y

N Is this the best to date?

Store Pattern
Three variations of this method have appeared, and in general order of increasing efficiency are:

a. Minimisation of waste (e.g. see reference 10).

b. Minimisation of Costs (ref.11)

c. A combination of (i) and (ii) ref.4).

N.Z. Forest Products Limited is using a heuristic reel program for certain small reel trim problems. (Ref.9) The program for this is very fast and has the big advantage that order tolerances can be specified as input. Trim loss is usually poorer than with the LP methods discussed later, but on certain problems it has definite advantages over an LP in spite of this.

It is unsatisfactory to choose the first good order combination that fulfills all the requirements, for this may, and usually does, result in poorer combinations at a later phase. Hence one has to consider all the orders of a run as a whole.

Since the order and machine constraints are linear and a linear cost equation can be formed, the problem seems suitable for linear programming.

However, even the ability of linear programming to look at all the given patterns together, and choose the best set, is not enough. For the number of possible patterns is so numerous that no computer is large enough to hold them all. Two approaches to overcome this problem have been tried.
Enter an initial feasible Soln. into the L.P. Basis

Solve this L.P. Problem

Collect the Simplex Multipliers

Solve a knapsack (i.e. Pattern) and form a new basis
2. Knapsack / Linear Programming Method

The first involves a series of linear programming pattern generation cycles, with each cycle converging upon the optimal solution. This model is as shown in Diagram 4.

The pattern generation phase is that of filling a knapsack and is achieved by dynamic programming. Recall that the simplex multipliers indicate the relative values that each constraint has on the objective. Thus the best knapsack can be filled by choosing those orders whose simplex multipliers most effect the objective function.

For sheeted trimming the filling of the knapsack becomes more complex.

This method is commonly known as the Gilmore/Gomory algorithm as these two men have done the majority of the research into it. See references (1,2,3.)

A reel trim program of this type written by Major and Fattal (Ref.8) for an IBM 1130 is now in regular use by the company. It is used for trimming almost half of the Kinleith reeled paper production. Trim loss on typical grades has been reduced from 1.5% to 0.5% of production, with worthwhile financial savings.

The program is very comprehensive, including many special features such as:

a. It can schedule over many paper machines simultaneously.
b. One can specify the production ratios for each machine.

c. One can assign specific orders to specific machines.

d. It can cut for and from stocks.

e. Orders can be assigned priorities, where low priority orders may not necessarily be satisfied.

f. It can schedule varying reel diameters and axle sizes.

Its Disadvantage Are:

a. It is very slow. Production problems generally take one to three hours to reach a satisfactory answer.

b. The non-integer answer are rounded to integers, and this sometimes causes problems with over or under-makes.

c. Setups are sometimes slightly greater than with a manual trim, but in general they are not so great as to cause production problems.

3. Heuristic Linear Programming Method

The other linear programming method consists of selecting a set of desirable patterns then conducting an L.P. on them to choose the best sub-set. We know of only one successful application using this idea, this being achieved by a British paper company. Before expanding this model the various approaches will be compared.
iv. **Comparison of Schedulers**

Firstly, since different models produce different economical amounts one cannot compare the costs, so percent trim loss is used.

Unfortunately we know of no sheeted trimmers using the Gilmore/Gomory approach in regular use. However, it is possible to compare two reel trim programs using the two L.P. methods.

For the Gilmore/Gomory program in use at NZFP a result of 0.75% waste was obtained on a large actual problem. Using the same data, the British program obtained 0.08%. It is expected that the same would be true for sheeted trimmers.

Both models do not have a strict control over minimum run length. In fact the main reasons for the disuse of the Gilmore/Gomory method for sheeted and reel trimming is that it takes too long, and splits up an order far too much. (Thus violating minimum run length).

A limited control is obtained in the British model since one can restrict the number of times each order is represented in the L.P. matrix. Heuristic Schedulers also offer a good control over run lengths.

Since the British model consists of heuristic and mathematical (L.P.) phases, we feel that the best hope for a good computer scheduler lies with this approach, and thus this model will be developed in more detail in the next section.
v. **Heuristic/L.P. Scheduler**

At present the theory of the topic paper "Two Dimensional Paper Trimming" that the Author prepared while at Victoria University (see Ref.7) is being developed in the form of a Fortran program for sheeted trimming.

As a secondary objective reel trimming is being restudied. The aim in reel trimming is to develop a faster program than the Gilmore/Gomory algorithm now in use, and one that is better than the heuristic program also in use.

The method of the heuristic/L.P. appears to offer the best approach because it can produce a near optimum answer and yet offer the advantages of the heuristic method.

1. **Sheeted Trimming**

   The raw data must contain the following:

   Length and width of the sheet, tonnage, percent over and undermaking tolerances and the grain direction for every order.

   The tonnage is converted to number of sheets, and by using the over and under percentages the lower and upper bounds can be computed, these being numbers of sheets. If an order allows grain switching then one must ensure that the rotated sheet is constrained between these limits.

   For stock cutting a subset of Common Stock sizes is included, and when generating patterns the possibility of cutting an order with this stock size is considered. The cost of stock cutting is of course added to this pattern cost.

   To use up stock one merely satisfies those orders
which exactly equal the stock dimensions. (This exact constraint is only partially justified since it is expensive to trim from stock with waste).

To illustrate such a model consider the simple example as shown in Diagram 5.

The set of patterns have been generated in a simple manner for there are many production constraints to consider. e.g. The number of slitter knives, and in this set certain patterns have been neglected e.g. the more complex combinations such as:

2 of 15 + 1 of 20 + 1 of 12

This was to simplify the example. However in the program this pattern would be considered. A fault in some heuristic programs is that they do not consider such combinations, and this could result in a poorer solution.

a. Pattern Generation

A comparison of two pattern generation programs has been done. The first program was one written by Hagan (see Ref.5) who has also experimented with a Gilmore/Gomory sheeted trim program (ref.6), it obtained 30 patterns compared with 76 patterns from the algorithm developed by the author (in Ref.7). Both programs used the same data and permitted multiple order combinations. It has not been ascertained why Hagans' generator obtained some, but all the possible combinations.

The pattern generator used is quite simple and is shown in diagram 7.

The program is designed to keep the best x patterns. For the example shown in Dig.5 the
patterns are entered into the L.P. matrix, also shown in Diagram 6. Thus these patterns become the activity or columns of the matrix.

If a Pi is to be 1 then we cut 10 inches of that pattern, i.e. an Area 70 x 10 square inches.

Hence if Pi was cut once then 4 sheets of 15 x 10 would result.

If P6 was cut once then 0.25 of 20 x 40 plus 0.5 of 40 by 20 would be obtained i.e. 0.75 or order 2.

Note that the stock reels have not been included in the matrix for these have no Constraints. If the storage was limited then stock production would be included with an upper bound.

The optimal solution for the example was to cut P4 11 times and P5 28.7. Obviously one cannot cut P5 as an non integer. However since the activities will be in thousands integerising is not serious.

To guarantee a feasible solution each order must be represented in a pattern by itself or with an unrestricted stock width.

The computer program that has been developed on the 1130 using this approach incorporates a simplex algorithm package. This is inefficient for solving L.P. problems with many Constraints and maybe changed to an improved technique at a later date.
### Order Data:

<table>
<thead>
<tr>
<th>Order</th>
<th>Width</th>
<th>Length</th>
<th>Lower Bounds</th>
<th>Upper Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>10</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>40</td>
<td>27</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>20</td>
<td></td>
<td>Order 2 can rotate</td>
</tr>
</tbody>
</table>

Stock Width: 12

**Paper Machine = 70**
**Cutter = 40**
**Max. Allowable Trim = 10**

### Pattern

1. 4 of 15
2. 3 of 15 + 1 of 20
3. 3 of 15 + 2 of 12
4. 2 of 15 + 1 of 40
5. 3 of 20
6. 1 of 20 + 1 of 40
7. 1 of 20 + 4 of 12
### DIAGRAM 6.

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.92</td>
<td>0.48</td>
<td>0.51</td>
<td>0.03</td>
<td>0.92</td>
<td>0.93</td>
<td>1.0</td>
</tr>
<tr>
<td>Cost Row</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 ≤</td>
<td>4.0</td>
<td>3.0</td>
<td>3.0</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27 ≤</td>
<td>0.25</td>
<td>0.5</td>
<td>0.75</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

£22
£33


**Diagram 7.**

**Widths:** \( W(i), i = 1, \ldots, M \)

1. Set \( t = 0 \), \( t \) denotes key order

2. \( t = t + 1 \)
   
   \[ A(t) = \frac{P.M. \text{ Width}}{W(t)} \]
   
   for \( t + 1 \leq i \leq M \) set \( A(i) = \frac{\text{Width} - \sum_{j} A(j) \cdot W(j)}{W(i)} \)

3. Loss = \( \text{Width} - \sum_{i} A(i) \cdot W(i) \)
   
   If loss \( \leq \) max. allowable trim, then accept pattern.

4. Find the largest \( i \) such that \( A(i) \neq 0 \).
   
   where \( t \leq i \leq m \)

5. \( A(s) = A(s) - 1 \)
   
   If \( S = M \) then go to 4.
   
   If \( S = t \) and \( A(t) = 0 \) then go to 2.

6. For \( S + 1 \leq i \leq m \) recalculate \( A(i) \)
   
   \[ A(i) = \frac{\text{Width} - \sum_{j} A(j) \cdot W(j)}{W(i)} \]
   
   go to 3.
Order Widths : \( W(i) \quad i=1,\ldots,M \)

1. Set \( t=0, \) \( t \) denotes key order

2. \( t=t+1 \)
   \[ A(t) = \text{P.M. Width} \frac{W(t)}{W(t)} \text"truncated integer". \]
   for \( t+1 \leq i \leq M \) set \( A(i) = \frac{\text{Width} - \sum_{t}^{m} A(j).W(j)}{W(i)} \)

3. Loss = Width - \( \sum_{t}^{m} A(i).W(i) \)
   If loss \( \leq \) max. allowable trim, then accept pattern.

4. Find the largest \( i = S \) such that \( A(i) \neq 0. \)
   where \( t \leq i \leq m \)

5. \( A(s) = A(s)-1 \)
   If \( S=M \) then go to 4.
   If \( S=\dagger \) and \( A(t) =0 \) then go to 2.

6. For \( S+1 \leq i \leq m \) recalculate \( A(i) \)
   \[ A(i) = \frac{\text{Width} - \sum_{t}^{m} A(j).W(j)}{W(i)} \]
   go to 3.
b. Costs

To cost a pattern the following values are added:

1. As before the cost of c times the waste. Dimensionally this is ($Q^{-1}$).

2. **Stock Cutting**: It can be shown that for optimal usage of a limited store the patterns should be costed by:

   \[ \lambda^* \text{ stock area used per ton (}$Q^{-1}$) \]

   Where \( \lambda = \text{largest ratio of } \frac{c'}{a_i} \)

   Where \( c' \) is the cost per ton of paper ($Q^{-1}$) and \( a_i \) is the storage area for one ton of stock. \( (L^{-1}) \)

   \[ \lambda \text{ is of dimension (}$L^{-2}$) i.e. \\
   \text{Cost of storage.} \]

3. **Cutter Costs**: For cutting, add the total cost of the duplex runs and the simplex runs. ($Q^{-1}$). This cost inclusion appears to have been neglected in most programs. It is hoped that by including it in the program described above it will provide better production schedules for the cutter crews.

c. Program Performance

Although the program is not fully developed and tested, indications are that it should prove successful for production use.

Firstly for a set of 10 orders, 3 were permitted to be rotated, and this reduced trim from 3 percent, to zero. This may
have been fortunate but nevertheless is an indication of the value of grain direction. It should be emphasised that grain direction has been neglected in all the sheeted programs published to date.

Secondly, the set of patterns was constrained to have a zero waste. This forced many patterns to make for stock, but the trim was again reduced from 3 percent to zero.

Due to the size limitation of the L.P. package the largest number of orders that the program can handle is 19. It is hoped to expand this to over 100 by using another L.P. package. A program will have to cope with about 120 orders to be useful in practice.

Consequently, for testing work to date the program is restricted to production runs with less than 20 orders.

These results are as follows:

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>Manual</th>
<th>Gilmore/Gomory</th>
<th>Heuristic</th>
<th>Heuristic/LP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>6.5</td>
<td>6.6</td>
<td>7.1</td>
<td>4.7</td>
</tr>
<tr>
<td>2.</td>
<td>?</td>
<td>-</td>
<td>2.0</td>
<td>0.4</td>
</tr>
<tr>
<td>3.</td>
<td>?</td>
<td>-</td>
<td>1.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

No. Orders

15
8
13

Note 1. That the Gilmore/Gomory program was unsatisfactory as it contained errors and did not cope with grain direction.

2. The number of Knife changes for the author's heuristic/LP model
There remains only one area of doubt. It is possible to obtain a pattern value that is too small. Setting these to zero may violate the order constraints. One cannot stipulate that every $p_i$ be greater than some minimum for this makes the L.P. infeasible.

Possible ideas around this are to introduce integer variables and a set up cost, but this would drastically slow the program and cut down its size. Another idea is to bias patterns so that large orders are cut with the next largest. Otherwise the final L.P. results could be patched up by some heuristic method. These ideas are being experimented with.

2. **Reeled Trimming**

As mentioned before a secondary program using the heuristic/L.P. method is being developed for reel trimming.

This approach is very similar to sheeted trimming except that there are no lengths, or grain direction.

The entries in the L.P. matrix are all integer and must never violate the upper bound (which contains the maximum number of reels of this order). If this value was greater than the upperbound then it becomes impossible for the L.P. phase to realise integer values.

*E.g.* Suppose the maximum number of reels of order size 15 inches width was 3. Then on a 70 inch paper machine the L.P. matrix would contain the pattern 3 of 15 rather than 4 of 15.
It is far more important to obtain integer answers in reeled trimming than it was for sheeted trimming. The Gilmore/Gomory reel trim program being used merely integerises the L.P. results and ensures that no constraints are violated. The result is still very close to the optimum but occasionally the integerised answers mean slight under or overmades of orders.

Program Performance

a. The following comparisons have been made:

<table>
<thead>
<tr>
<th>Manual</th>
<th>Heuristic</th>
<th>Heuristic/L.P.</th>
<th>No. Orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>0.0</td>
<td>11</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>1.8</td>
<td>0.2</td>
<td>0.1</td>
<td>10</td>
</tr>
</tbody>
</table>

Now the heuristic/L.P. program integerises the activity values, consequently values are occasionally obtained that violate the Constraints.

A possible method to overcome this is to specify a set of tighter lower and upper bounds than those used manually and hope that the integer solution of the L.P. still lies within these looser bounds.

For both reeled and sheeted trimming the output order is important since each pattern change means a series of knife changes. One can solve the problem of minimising the total knife changing times, and thus determine the output order, for this is an example of the travelling salesman problem. However, this has not been attempted as there are other production considerations which would sometimes invalidate the sequence.

Conclusion

The best approach for computerising sheeted trim production appears to be with a heuristic/L.P. model. It is envisaged that the problems of implementing such a program
will be less than those encountered for reels, since computerised reel trimming has been accepted as a success.

As the situation becomes more complex e.g. installation of wider paper machines, then computerised paper scheduling will provide even greater gains.

Acknowledgements

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REFERENCES


