LINEARLY COMPACT ALGEBRAS AND COALGEBRAS

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Abstract. We specify a broad class of varieties of algebras for which necessary and sufficient condition can be obtained, for the validity of the so-called “Fundamental Theorem on Coalgebras”. The local structure of linearly compact algebras of these varieties are also considered.

The theory of coalgebras was initially developed as a part of the structure theory of Hopf algebras which have appeared in topology in various ways. Hopf algebras were the first examples of bialgebras studied by algebraists and they were essentially associative objects with associative multiplication and comultiplication (see [1] and the recent monograph [2] devoted to this theory). Lie coalgebras having associative coenveloping coalgebras were considered in [3, 4, 5, 6]. Michaelis [7,8] considered Lie coalgebras from the abstract point of view, and he discovered that not all of them have associative coenveloping coalgebras, but only locally finite ones. Lie bialgebras were introduced by V. Drinfel’d [9] for the sake of quantum groups.

Since then it became clear that every variety of algebras has a corresponding category of coalgebras. For coalgebras of an arbitrary variety all necessary definitions were first given by J. Anquela, T. Cortés and F. Montaner [10], where basic results of the associative theory were generalized to coalgebras of an arbitrary variety. But one of the most important results, the so called “Fundamental Theorem on Coalgebras”, which states that every associative coalgebra is locally finite, cannot be generalized to coalgebras of an arbitrary variety. In fact, Michaelis showed that it is not true for Lie coalgebras. Nevertheless, J. Anquela, T. Cortés and F. Montaner proved that “Fundamental Theorem on Coalgebras” is true for alternative and Jordan coalgebras. After this result was delivered at the 3rd International Conference on Nonassociative Algebras and Their Applications, E. Taft, in a private conversation with the author expressed an opinion that it would be interesting to find some necessary and sufficient conditions on the variety under which all coalgebras of this variety are locally finite. I. Shestakov opined that these conditions must be similar to those (see [11]) under which the locally nilpotent radical exists. Partially motivated by this question the author suggested in [12] a necessary and sufficient condition for a Lie coalgebra $M$ to be locally finite. It was given in terms of its dual Lie algebra $L$, which is a linearly compact topological algebra with respect to the linear weak topology. It was shown that local finiteness of $M$ is equivalent to the existence of a base of ideal neighborhoods of zero of $L$ which is, in turn, equivalent to $L$ being topologically algebraic.

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In this paper we shall specify a broad class of varieties of algebras for which necessary and sufficient condition can be obtained, similar to the condition for Lie algebras in [12]. As varieties of Jordan and alternative algebras are contained in the specified class of varieties and since the necessary and sufficient conditions mentioned become trivial for them, the result can be considered as a generalization of the result of J. Anquela, T. Cortés and F. Montaner [10]. On the other hand the results on the local structure of linearly compact alternative and Jordan algebras are also new. Due to this connection with coalgebras the results on the structure of linearly compact algebras are getting more important. The reader may notice their similarity to the corresponding results on the structure of compact rings [13,14].

1. Coalgebras and their Dual Algebras

Let $k$ be a field. A coalgebra over $k$ is a vector space $C$ over $k$ equipped with an arbitrary $k$-linear map $\Delta : C \to C \otimes C$. This mapping $\Delta$ is called the diagonal mapping or comultiplication of the coalgebra $C$. In practice this comultiplication obeys certain “identities”, so that we can speak of associative, Lie, and other coalgebras. Although it is possible to give this definitions directly in each particular case the general rule will be hidden. It comes to the surface with the definition of the dual algebra of a coalgebra.

Let’s consider the dual space $A = C^*$ of $C$ and define a multiplication on it by the formula

$$(f \cdot g)(c) = (f \otimes g)(\Delta c),$$

or in Sweedler’s notations [1], if $\Delta c = \sum_c c_1 \otimes c_2$, then

$$(f \cdot g)(c) = \sum_c f(c_1)g(c_2).$$

**Definition 1.** Let $\mathcal{M}$ be an arbitrary variety of algebras over $k$. A coalgebra $C$ is called a coalgebra of the variety $\mathcal{M}$ if its dual algebra $A = C^*$ is an algebra of $\mathcal{M}$.

As it is well-known [15], $A$ as a vector space has a natural topology on it, which is defined by the neighborhood base of zero consisting of the subspaces

$$V^\perp = \{f \in A \mid f(V) = 0\}, \quad \dim V < \infty.$$ 

It is linear in a sense that it possesses a neighborhood base of zero consisting of subspaces and it is called the linear weak topology or the weak-* topology. Let’s recall also [16] that a linear topology of a vector space $E$ is said to be linearly compact if any filter base on $E$ consisting of closed linear varieties has a non-empty intersection.

**Proposition 1.** With the linear weak topology $A$ is a linearly compact topological algebra.

**Proof.** Given that $\Delta c = \sum_c c_1 \otimes c_2$, we define the subspaces $L(c)$ and $R(c)$ to be the spans of all $c_1$’s and $c_2$’s, respectively.

Suppose that $V^\perp$ for $V = \{v_1, \ldots, v_k\}$ is one of the neighborhoods from the base. Set $U = L(v_1) + \cdots + L(v_k)$ and $W = R(v_1) + \cdots + R(v_k)$. Then (2) implies that

$$U^\perp A \subseteq V^\perp, \quad AW^\perp \subseteq V^\perp.$$
It shows the continuity of the multiplication of \( A \). The proof of linear compactness of \( A \) stems from the isomorphism of the vector space of \( A \) with the direct topological product \( k^d \), where \( d \) is the dimension of \( A \) (for more details see [15, p. 97]).

**Definition 2.** A subspace \( B \subseteq C \) is said to be a subcoalgebra of \( C \) if \( \Delta(B) \subseteq B \otimes B \). A coalgebra \( C \) is locally finite if any \( c \in C \) lies in some finite dimensional subcoalgebra. As the sum of subcoalgebras is a subcoalgebra, it is equivalent to the condition that any finite number of elements \( c_1, \ldots, c_k \in C \) lie in some finite dimensional subcoalgebra.

It is known that all associative coalgebras are locally finite [1]. This theorem is of great importance for the theory of associative coalgebras and it is called the "Fundamental Theorem on Coalgebras". Local finiteness is also an important property for Lie coalgebras (although not always fulfilled) as it is exactly for the class of locally finite Lie coalgebras that a dual Poincaré-Birkhoff-Witt Theorem holds [8].

If \( F \subseteq A \), we also denote \( F^\perp = \{ c \in C \mid f(c) = 0 \text{ for all } f \in F \} \). Note that the sign \( \perp \) here has a slightly different meaning than before.

**Proposition 2.** If \( S \) is a finite dimensional subcoalgebra in \( C \), then \( S^\perp \) is an ideal of \( A \) of finite codimension. If \( I \) is an ideal of \( A \) of finite codimension, then \( I^\perp \) is a finite dimensional subcoalgebra of \( C \).

**Proof.** The proof is straightforward and immediately follows from (2).

**Proposition 3.** \( C \) is locally finite if and only if \( A \) has a basis of neighborhoods of zero consisting of ideals of finite codimension.

**Proof.** Suppose that \( C \) is locally finite. Consider a neighborhood of zero \( V^\perp \), \( \dim V < \infty \), from the base of neighborhoods of zero in \( A \). Then the finite basis of \( V \)—and hence \( V \) as a whole—is contained in a finite dimensional subcoalgebra \( W \) of \( C \) and therefore \( W^\perp \subseteq V^\perp \) is an ideal neighborhood of \( A \), which lies in \( V^\perp \).

Suppose now that \( A \) has a basis of neighborhoods of zero consisting of ideals. As these ideals are open, they all have finite codimensions. For any \( x \in C \) the set \( x^\perp \) is open and there exists an ideal \( I \) from this basis such that \( I \subseteq x^\perp \). As \( \text{codim} \ I < \infty \), subcoalgebra \( I^\perp \) is finite dimensional and \( x \in I^\perp \).

2. Admissible Varieties of Algebras. Topological Restrictedness

Let \( A \) be an arbitrary algebra over \( k \), which we assume to be of characteristic \( \neq 2 \). For every element \( a \in A \) two mappings from the algebra to itself are defined:

\[
R_a : x \mapsto xa \quad L_a : x \mapsto ax .
\]

These endomorphisms of the vector space of the algebra \( A \) are called the operators of right and left multiplication by the element \( a \), respectively. The subalgebra of the algebra of endomorphisms \( \text{End}_k(A^d) \) of the vector space \( A^d \) of the algebra \( A \) generated by all possible operators \( R_a \) and \( L_a \), where \( a \in A \), is called the multiplication algebra of the algebra \( A \) and is denoted by \( M(A) \). Elements of \( M(A) \) will be considered as acting on elements of \( A \) from the left, for example:

\[
R_aL_bR_cx = (b(xc))a .
\]
The identities satisfied by algebras imply a series of relations for the multiplication operators. It is possible also to define identities by means of these relations. We shall list some of them which we shall refer to later. The general reference for them will be \([17, 18]\).

In any Jordan algebra \(A\) for arbitrary \(x, y \in A\) the following relations are valid:

\[
R_x = L_x, \tag{2}
\]
\[
2R_x R_y R_x = -R_x^2 y + 2R_x R_{xy} + R_y R_{x^2}. \tag{3}
\]

We will convert them to a form

\[
R_x \equiv L_x, \tag{4}
\]
\[
R_x R_y R_x \equiv 0, \tag{5}
\]

where the equivalence means equality modulo the linear combination of operator words of operator length smaller than that of the left hand side, i.e. containing fewer operators of right and left multiplications.

In any Mal'tsev algebra \(A\) for arbitrary \(x, y \in A\) the following relations are valid:

\[
R_x = -L_x, \tag{7}
\]
\[
R_x R_y R_x = R_x R_x R_y + R_{(xy)} x + R_{xy} R_x, \tag{8}
\]

for which the corresponding equivalences will be

\[
R_x \equiv -L_x, \tag{9}
\]
\[
R_x R_y R_x \equiv R_x R_x R_y, \tag{10}
\]

For an alternative algebra \(A\) for arbitrary \(a, b \in A\)

\[
R_a R_b = -R_b R_a + R_{ab+ba},
\]
\[
L_a L_b = -L_b L_a + L_{ab+ba},
\]
\[
L_a R_b = R_b L_a + R_b R_a - R_{ab},
\]
\[
R_a L_b = L_b R_a + R_b R_a - R_{ab}.
\]

from which the following relations can be derived:

\[
R_x R_y R_x \equiv R_x L_y R_x \equiv L_x L_y L_x \equiv L_x R_y L_x \equiv 0. \tag{11}
\]

\[
R_x R_y L_x \equiv R_x L_x R_y, \tag{12}
\]
\[
R_x L_y L_x \equiv -R_x L_x L_y, \tag{13}
\]
\[
L_x L_y R_x \equiv L_x R_x L_y, \tag{14}
\]
\[
L_x R_y R_x \equiv -L_x R_x R_y. \tag{15}
\]
Generalizing we can proceed with the following definition which will embrace these three important cases. For convenience of notations we will denote also $M^+_x = R_x$ and $M^-_x = L_x$, so that an arbitrary operator can be denoted as $M^\varepsilon_x$, where $\varepsilon \in \{+, -\}$ or $M^\varepsilon(x)$, especially if $x$ itself is an expression containing indices.

**Definition 3.** We shall say that a homogeneous variety of algebras $\mathcal{M}$ is admissible if for an arbitrary algebra $A$ of $\mathcal{M}$ and for all $x, y \in A$ all eight operator products of the type $M^{\varepsilon_1}_x M^{\varepsilon_2}_y M^{\varepsilon_3}_x$, where $\varepsilon_i \in \{+, -\}$, are expressible in the following two ways:

(a) as linear combinations of operator products of the type $M^{\tau_1}_x M^{\tau_2}_x M^{\tau_3}_y$, $\tau_i \in \{+, -\}$, and products of smaller operator length:

$$M^{\varepsilon_1}_x M^{\varepsilon_2}_y M^{\varepsilon_3}_x = \sum_{\tau} \alpha_{\tau} M^{\tau_1}_x M^{\tau_2}_y M^{\tau_3}_x,$$

where $\alpha_{\tau} \in k$,

(b) as linear combinations of operator products of the type $M^{\tau_1}_y M^{\tau_2}_x M^{\tau_3}_x$, $\tau_i \in \{+, -\}$, and products of smaller operator length:

$$M^{\varepsilon_1}_x M^{\varepsilon_2}_y M^{\varepsilon_3}_x = \sum_{\tau} \beta_{\tau} M^{\tau_1}_y M^{\tau_2}_x M^{\tau_3}_x,$$

where $\beta_{\tau} \in k$.

Relations (5) – (6), (9) – (10), (11) – (15) mean that varieties of Jordan, Mal’tsev and alternative algebras are admissible. Note that (16) and (17) are, indeed, the same as in [11].

Suppose now that $A$ is a topological algebra. In this case the multiplication algebra can be also endowed with a topology by defining the base of neighborhoods of zero consisting of the subsets

$$\Psi(U) = \{ \omega \in M(A) \mid \omega A \subseteq U \},$$

where $U$ is a neighborhood of zero in $A$. This topology clearly converts the multiplication algebra into a topological algebra, and we assume further that the multiplication algebra is always topologized in this manner. Of course, this topology can be discrete for some algebras but not for linearly compact ones as we shall see from the following lemma. We can prove it without any use of identities for arbitrary nonassociative algebras.

**Lemma 1.** Let $A$ be an arbitrary linearly compact algebra over a field $k$ and let $U$ be an arbitrary neighborhood of zero in $A$. Then there exists a neighborhood of zero $W$ such that $WA \subseteq U$ and $AW \subseteq U$.

**Proof.** It is sufficient to prove only the first inclusion. As the multiplication in $A$ is continuous there exists a neighborhood of zero $V$ such that $V^2 \subseteq U$. Since $V$ has a finite codimension we can write $A = ka_1 + \ldots + ka_m + V$. There exist also neighborhoods $V_1, \ldots, V_m$ such that $V_i a_i \subseteq U$ for $i = 1, \ldots, m$. It is clear now that for the neighborhood $W = V \cap V_1 \cap \ldots \cap V_m$

$$WA = Wa_1 + \ldots + Wa_m + WV \subseteq V_1 a_1 + \ldots + V_m a_m + V^2 \subseteq U,$$
and the lemma is proved.

Let $A$ be an algebra and $M(A)$ be its multiplication algebra. By $M_n(A)$ we shall denote the subspace of $M(A)$ spanned by all operator products of operator length less than or equal to $n$.

**Corollary 1.** Let $A$ be an arbitrary linearly compact algebra, let $U$ be a neighborhood of zero in $A$ and $n$ be an integer. Then there exists a neighborhood $W = W(n, U)$ such that

$$M_n(A)W \subseteq U.$$ 

**Proof.** It follows from the lemma by an easy induction.

This lemma also implies that for an arbitrary linearly compact algebra $A$ the topology of its multiplication algebra $M(A)$ is not discrete and the regular right and left representations

$$R : x \mapsto Rx \quad L : x \mapsto Lx$$

are continuous.

**Definition 4.** We shall say that a topological algebra $A$ is topologically restricted if for an arbitrary element $x \in A$ and an arbitrary neighborhood of zero $W$ of $M(A)$ there exists an integer $n = n(x, W)$ such that for all $\varepsilon_1, \ldots, \varepsilon_n \in \{+, -\}$

$$M_{\varepsilon_1}^x \cdots M_{\varepsilon_n}^x \equiv \omega \in W.$$ (18)

Note that being topologically restricted is not a restriction at all for Jordan and alternative algebras. For Jordan algebras it follows from (4) that

$$R_x R_x R_x \equiv 0.$$ 

One should also remember that Jordan algebras are commutative ones. Therefore $n = 3$ will always do the job. The same is true for alternative algebras. It follows from relations (11) - (15) since

$$R_x R_x \equiv 0, \quad L_x L_x \equiv 0.$$ 

But, of course, it is a restriction, say for Lie algebras.

**Proposition 4.** If a topological algebra $A$ has a neighborhood base of zero consisting of ideals of finite codimension, then it is topologically restricted.

**Proof.** Let $\Psi(U)$ be a neighborhood of zero from the base of $M(A)$. We can assume that $U$ is an ideal neighborhood of $A$. Let $\overline{A} = A/U$. Let $x \in A$ be an arbitrary and $\bar{x} = x + U$. Then, as $\overline{A}$ is finite dimensional, $M(\overline{A})$ is also finite dimensional and there exists an integer $n$ such that

$$M_{\varepsilon_1}^x \cdots M_{\varepsilon_n}^x \equiv 0,$$

or, what is the same,

$$M_{\varepsilon_1}^x \cdots M_{\varepsilon_n}^x \equiv \omega \in \Psi(U),$$

and $A$ is topologically restricted.

This proposition explains our interest towards the condition of being topologically restricted since it is a necessary condition for having a neighborhood base of ideals of finite codimension, which is important to us in view of Proposition 3. We shall prove that for linearly compact algebras of admissible variety this condition is also sufficient.
3. The Local Structure of Linearly Compact Algebras of Admissible Varieties

We shall start with the following combinatorial lemma. For brevity we shall refer to algebras of admissible varieties as to admissible algebras.

Lemma 2. Let $A$ be an admissible linearly compact topologically restricted algebra over a field $k$ and $U$ be a neighborhood of zero in $A$. Then there exists an integer $n = n(U)$ such that

$$M(A) = M_n(A) + \Psi(U). \tag{19}$$

Proof. Let $W$ be a neighborhood, which by Lemma 1 satisfies $WA \subseteq U$ and $AW \subseteq U$. Intersecting $W$ with $U$, if necessary, we can assume that $W \subseteq U$. As $A$ is linearly compact, this neighborhood is a linear subspace of a finite codimension. Thus $A/W = \{\overline{a}_1, \ldots, \overline{a}_m\}$, $\overline{a}_i = a_i + W$, for elements of some finite subset $B = \{a_1, \ldots, a_m\}$ of $A$. Suppose that $p_i$ is an integer such that for arbitrary $\varepsilon_1, \ldots, \varepsilon_{p_i} \in \{+, -\}$

$$M^{\varepsilon_1}(a_i) \ldots M^{\varepsilon_{p_i}}(a_i) \equiv \omega_i \in \Psi(W). \tag{20}$$

Such integers $p_i$ exist since $A$ is topologically restricted. Set now $n = 2m(p - 1)$, where $p$ is the maximal integer among $p_1, \ldots, p_m$. We shall prove that this integer satisfies the desired property.

It is sufficient to show that an arbitrary operator product

$$\eta = M^{\varepsilon_1}(x_1) \ldots M^{\varepsilon_t}(x_t) \tag{21}$$

is contained in the set $M_n(A) + \Psi(U)$. Since it is clear for $t \leq n$ we can proceed by induction on $t$.

For any $j = 1, \ldots, t$ we can write $x_j = a_{ij} + v_j$, where $a_{ij} \in B, v_j \in W$. Thereby we can suppose in (21) without loss of generality that $x_j \in B \cup W$.

Linearization of (16) gives us

$$M^{\varepsilon_1}_x M^{\varepsilon_2}_y M^{\varepsilon_3}_z \equiv -M^{\varepsilon_1}_y M^{\varepsilon_2}_x M^{\varepsilon_3}_z + \sum_\tau \alpha_\tau M^{\tau_1}_x M^{\tau_2}_y M^{\tau_3}_z + \sum_\tau \alpha_\tau M^{\tau_1}_z M^{\tau_2}_x M^{\tau_3}_y. \tag{22}$$

This means that we can move operators $L_z$ and $R_z$ to the left modulo words of smaller operator length.

Suppose that for some $i$ the total amount of operators $R_{a_i}$ and $L_{a_i}$ in the operator word $\eta$ is $p$ or more. Using (22) let's move them all to the left until all subwords $M^{\varepsilon_1}(x)M^{\varepsilon_2}(y)M^{\varepsilon_3}(a_i)$, where $x \neq a_i$ and $y \neq a_i$, will be eliminated. Then there might be some subwords of the type $M^{\varepsilon_1}(a_i)M^{\varepsilon_2}(y)M^{\varepsilon_3}(a_i)$, which can be eliminated by (16) and again (22). At the end of this process we will gather all $R_{a_i}$ and $L_{a_i}$ on the left, starting at the first or the second position of the operator word. Therefore we shall represent $\eta$ in one of the two forms:

$$\eta \equiv M^{\varepsilon_1}(a_i) \ldots M^{\varepsilon_p}(a_i) M^{\tau_1}(x_1) \ldots M^{\tau_q}(x_q), \tag{23}$$

or

$$\eta \equiv M^{\tau_1}(x_1) M^{\varepsilon_1}(a_i) \ldots M^{\varepsilon_p}(a_i) M^{\tau_2}(x_2) \ldots M^{\tau_q}(x_q), \tag{24}$$
where \( x_i \in B \cup W \) and \( p + q = t \). Due to (20) in the first case

\[
\eta \equiv \omega_1 M^{T_1}(x_1) \cdots M^{T_q}(x_q) \in \Psi(W) \subseteq \Psi(U),
\]

and in the second

\[
\eta \equiv M^{T_1}(x_1) \omega_1 M^{T_2}(x_2) \cdots M^{T_q}(x_q) \in M^{T_1}(x_1)\Psi(W) \subseteq \Psi(U).
\]

Suppose now that for all \( i \) the total amount of operators \( R_{a_i} \) and \( L_{a_i} \) in the operator word \( \eta \) is less than \( p \). Linearizing (17) we obtain

\[
M_{x}^{\varepsilon_1} M_{y}^{\varepsilon_2} M_{z}^{\varepsilon_3} = -M_{x}^{\varepsilon_1} M_{y}^{\varepsilon_2} M_{z}^{\varepsilon_3} + \sum_\tau \beta_\tau M_{y}^{T_{1\tau}} M_{x}^{T_{2\tau}} M_{z}^{T_{3\tau}} + \sum_\tau \beta_\tau M_{y}^{T_{1\tau}} M_{z}^{T_{2\tau}} M_{x}^{T_{3\tau}}. \tag{27}
\]

Using (27) we can eliminate from \( \eta \) all subwords \( M_{x}^{\varepsilon_1}(a_i) M_{y}^{\varepsilon_2}(w_1) M_{z}^{\varepsilon_3}(w_2) \) moving now \( a_i \)'s to the right. After elimination of these subwords \( \eta \) will be equivalent either to an operator word starting with an operator of right or left multiplication by an element of \( W \) or to the word containing of no more than \( 2m(p - 1) = n \) operators. In the first case \( \eta \) is equivalent to an operator word from \( \Psi(W) \subseteq \Psi(U) \), while in the second case it is congruent to the word from \( M_n(A) \). Lemma is proved.

**Theorem.** Every admissible linearly compact topologically restricted algebra \( A \) has a basis of neighborhoods consisting of ideals (of finite codimension).

**Proof.** Let \( U \) be an arbitrary neighborhood of zero of \( A \). Then by Lemma 2 there exists an integer \( n \) for which

\[
M(A) = M_n(A) + \Psi(U).
\]

By Corollary 1 there exists also a neighborhood \( V \), which is contained in \( U \) and such that

\[
M_n(A)V \subseteq U.
\]

The ideal of \( A \) generated by \( V \) is given by

\[
\langle V \rangle = V + M(A)V = V + M_n(A)V + \Psi(U)V \subseteq U,
\]

and it is open since it contains \( V \). This proves the theorem.

**Corollary 2.** Every linearly compact simple alternative, Jordan or algebraic Mal'tsev algebra is finite dimensional and discrete.

**Corollary 3.** Let \( A \) be a linearly compact alternative or Jordan algebra. Then its (quasiregular) radical \( J(A) \) is topologically nilpotent, i.e. for an arbitrary neighborhood of zero \( U \) there exists an integer \( n = n(U) \) such that

\[
J(A)^n \subseteq U.
\]

**Proof.** We can find an ideal neighborhood \( W \) inside every neighborhood of zero \( U \). Since every open subset of \( A \) is a subspace of finite codimension, the quotient algebra \( \overline{A} = A/W \) is finite dimensional, and its radical is nilpotent: \( J(\overline{A})^n = \overline{0} \). As the radical \( J(A) \) is in the inverse image of \( J(\overline{A}) \) we have \( J(A)^n \subseteq U \).
4. Local Finiteness of Coalgebras

In this short section we can finally formulate our main result concerning coalgebras and formulate a necessary and sufficient condition for the coalgebras of admissible variety to be locally finite.

**Theorem 4.1.** A coalgebra $C$ of an admissible variety is locally finite if and only if its dual algebra $A = C^*$ is topologically restricted.

**Proof.** Immediately follows from Theorem 1 and Propositions 3 and 4.

**Corollary 4.** [Anquela-Cortés-Montaner]. Every alternative or Jordan coalgebra is locally finite.

**References**