ABSTRACTS OF PAPERS

The Tenth New Zealand Mathematics Colloquium held at
Otago University, Dunedin, 19-22 May, 1975

ALGEBRA AND LOGIC

K. ASHTON, University of Auckland. Algebra-like structures and
biological structure.

The problem is to construct a system sufficiently normal to per­mit the use of familiar methods of universal algebra, yet able to
model the complexities of biological form. In particular the system
must handle not only partialness but also functions whose domains are
not Cartesian products. We cannot assume a stratified system but
must accept differing lengths of chains of components. We shall
also need to compare structures of differing "algebraic type" in
order to simplify detail and in order to consider morphogenetic and
other changes in time. A component set is the range of some func­tion, arguments of that function being sub-components. A level
consists of a set of component sets, each of the component sets
acting as an argument set for a function whose range lies in a
higher level. The component sets are indexed with regard to the
functions in which they appear as arguments as in Higgins [2].
The actual tuples in the domain of a given function are defined by
means of a context-free language. (This language may be presented
as context sensitive with contexts occupied by constructs from other
levels and thus acting as a control mechanism.) This complex of
indexed sets and "control" languages constitutes the device for
regaining a formal algebraic fullness from a situation essentially
partial. Biologically speaking, it represents the genetic and
external constraints guiding the formation of the structure. The
levels form an upper semi-lattice, called the skeleton, a node a
covering nodes \( \hat{b}_1, \cdots, \hat{b}_n \) exactly when level \( a \) consists of argu­ment sets \( A_1, \cdots, A_n \), the members of \( A_1 \) being constructed from
the argument sets in level \( b_1 \). The skeleton, together with the
indexing system constitute the "type" of the structure. Two
structures \( S_1 \) and \( S_2 \) are comparable iff there exists a structure
\( S \) whose skeleton and indexing system are homomorphs of the corres-
ponding objects of $S_1$ and $S_2$. Structure homomorphisms, congruences and direct products are definable within a class of structures of the same type in essentially the usual manner. A given structure $S$ defines a set of possible, similar organisms. A congruence on $S$ classifies these organisms. A postulated classifier can be tested for consistency with the structure and converted to its "best possible form" using the formal properties of structure congruences. Possible populations of a given organism can be generated by a substructure of a direct power of the structure defining the organism and communities of organisms of the same type by a similar use of a direct product. A hierarchical classification of organisms of dissimilar types can be approached in terms of the comparison of structures. Comparison can also be used in considering a morphogenetic, phylogenetic or seral sequence, the successive unfolding of the "genetic" base of the structure. As in universal algebra, within a given type, the class of all homomorphs of all substructures of all direct products of a class of structures is closed under these same three operations and represents the variety of forms possible within that type.

REFERENCES
1. K. Ashton, Department of Mathematics, University of Auckland, Report Series No.8,13,16 and 32.
2. P. J. Higgins, Algebras with a Scheme of Operators.

M. J. BROCKWAY, Victoria University of Wellington. A generalization of the Boolean filter concept.

The work described in this paper was motivated by some work I was doing in Boolean valued model theory; in particular, a search for a Boolean valued analogue of the completeness theorem. The paper begins with the observation that if $P$ is a Boolean algebra, a subset $F$ of $P$ is a filter if and only if its characteristic function is a map from $P$ into 2 (the 2-element Boolean algebra) which preserves meets; $F$ is a proper filter if and only if its characteristic function preserves meets and zero; $F$ is an ultrafilter if and only if its characteristic function preserves meets, zero and complements (i.e. is a homomorphism of $P$ into 2). If $Q$ is a complete Boolean algebra, a map $F$ from $P$ into $Q$ is called a $Q$-filter if and only if it preserves meets, a proper $Q$-filter if and only if it preserves meets and zero, and a $Q$-ultrafilter if and only if it is a homomorphism. These constructions have properties of which the standard properties of filters are special cases, and there is also a theory of bases and sub-bases of $Q$-filters and a generalization of the finite intersection property which applies to any map from $P$ into $Q$. 
J. CLARK, University of Otago. Coherent Rings.

Let \( R \) be a ring with identity. A (left) ideal \( I \) of \( R \) is called \textit{finitely presented} if there exists a short exact sequence of (left) \( R \)-modules of the form \( 0 \to K \to F \to I \to 0 \) where \( F \) is a finitely generated free \( R \)-module and \( K \) is a finitely generated submodule of \( F \). From the definition, every finitely presented ideal must be finitely generated. The converse of this is false in general. The ring \( R \) is called (left) \textit{coherent} if every finitely generated (left) ideal of \( R \) is finitely presented. Chase (Theorem 2.2) has shown that the ring \( R \) is coherent if and only if the intersection of any two finitely generated ideals of \( R \) is finitely generated, and the annihilator of any element of \( R \) is also finitely generated. It follows that Noetherian rings, valuation domains, and von Neumann regular rings are examples of coherent rings. Indeed coherent rings are often studied from the point of view that they generalise Noetherian rings. In this paper we compare the stability properties of coherent and Noetherian rings.

REFERENCE

L. J. CUMMINGS, University of Newcastle, Australia, and University of Waterloo, Canada. Cyclic Symmetry Classes.

Symmetry classes of tensors are constructed in the same way that the Grassmann space, or \( p \)th exterior power, is constructed from the underlying vector space. They were introduced by H. Weyl to represent the general linear group. If \( V \) is a finite-dimensional complex vector space let \( \mathbb{S}^mV \) be the space of \( m \) contravariant tensors over \( V \). One definition takes the range of a symmetry operator \( S: \mathbb{S}^mV \to \mathbb{S}^mV \) given by \( S = \frac{1}{|G|} \sum_{\pi \in G} \chi(\pi)P(\pi) \) as the symmetry class \( \chi \langle G \rangle \) where \( G \) is a subgroup of \( S_m \), \( \chi: G \to \mathbb{C}^* \) is a linear character, and \( P(\pi): \mathbb{S}^mV \to \mathbb{S}^mV \) is the linear mapping sending \( x_1 \otimes \cdots \otimes x_m \) in \( \mathbb{S}^mV \) to \( x_{\pi^{-1}(1)} \otimes \cdots \otimes x_{\pi^{-1}(m)} \). We show that most symmetry classes behave like the Grassmann space \( (G=S_m, \chi=\varepsilon, \text{the alternating character}) \) in the sense that when \( m \) is fixed and the dimension of the underlying vector space is
increased, the resulting symmetry classes can vanish at only finitely many values of the dimension. We exhibit simple combinatorial formulae for the dimension of $V(G)$ in case $G$ is a cyclic group generated by an $m$-cycle.

REFERENCES


M. J. CURRAN, University of Auckland. Decomposable involution centralizers.

Let $G$ be a finite group with an involution $t$. We say $t$ is central if $t$ is in the centre of a Sylow 2-subgroup of $G$.

Suppose $G$ satisfies the following hypothesis: $G$ has a central involution $t$ whose centralizer in $G$ has the structure $C(t) = \langle t \rangle \times F$ where $F$ is a non-abelian simple group. Janko [1] has shown that if $G$ has no subgroup of index 2, and if $F \cong A_5$, then $G \cong J_4$, the Janko simple group of order 175,560; and Janko and Thompson [2] have proved that if $F \cong PSL(2,q), q \equiv 3,5 (mod 8), q > 5$, then $q = 2^{2k+1}(n\geq 1)$ and $G$ is simple (these are the groups of Ree type). Roughly speaking, we show these are the only simple groups with a centralizer of the above form. Specifically, we prove the following. THEOREM: Let $G$ satisfy the above hypothesis, where $F$ is isomorphic to any alternating simple group $A_n (n\geq 6)$, or any classical or exceptional Lie type simple group of odd characteristic (except $PSL(2,q), q \equiv 3,5 (mod 8), q > 5$).

Then $G$ has a (normal) subgroup of index 2 not containing $t$.
In particular $G$ is not simple.

REFERENCES


R. I. GOLDBLATT, Victoria University of Wellington. First order definability in modal logic.

A class of structures for a first order language is elementary if it is the class of all models of some first order sentence. The structures that provide models for propositional modal logic consist of a binary relation on a set, and so are also appropriate for the first order language, with equality, of a single dyadic predicate. PROPOSITION 1: Let $X$ be a modal-axiomatic class of structures (i.e. $X$ is the class of all models of some set of modal sentences).
Then (a) \( X \) is elementary iff \( X \) is a union of elementary classes (\( E \)-elementary). (b) \( X \) is an intersection of elementary classes (\( A \)-elementary) iff \( X \) is closed under ultrapowers iff \( X \) is closed under ultraproducts iff \( X \) is closed under the relation of first order semantic equivalence of models (\( \Sigma \Delta \)-elementary).

**PROPOSITION 2:** There exists a modal-axiomatic class that is \( \Delta \)-elementary but not elementary. **PROPOSITION 3:** If \( X \) is the class of models of a single modal sentence, then all six conditions in Proposition 1 are equivalent.

M. D. HENDY, Massey University. *Testing for units in a pure cubic field.*

Let \( Q(\alpha) \) be an algebraic extension over \( \mathbb{Q} \), of degree \( n = s + 2t \), where \( s \) and \( 2t \) are the numbers of real and complex roots of the minimal polynomial of \( \alpha \). In order to analyse the multiplicative structure of the algebraic integers it is necessary to know the set of units, and the ideal class number. According to a well known theorem of Dirichlet, every integral unit \( \mu \) of \( Q(\alpha) \) can be expressed in the form \( \mu = \xi \epsilon_1^{a_1} \cdots \epsilon_r^{a_r} \) where \( \xi \) is a root of unity in \( Q(\alpha) \), and \( \epsilon_i \) are \( r = s + t - 1 \) units \( 0 < \epsilon_i < 1 \) known as the fundamental units. For complex quadratic fields \( r = 0 \), and the problem is simple. For real quadratic fields \( r = 1 \), and the fundamental unit is readily obtainable from the continued fraction expansion of \( \sqrt{d} \). In the next case examined, the pure cubic, \( s = t = 1 \), again \( r = 1 \). However the analogues of continued fraction algorithms in dimensions higher than two do not always generate the fundamental unit. In this paper, we examine a device whereby we can determine explicitly whether or not a given real number \( \mu \) is a unit of the field under examination. Using this test we can examine the numbers \( \mu^{1/p} \), for small primes \( p \), where \( \mu \) is a suspected fundamental unit. From this test we can explicitly determine the fundamental unit, given any unit.

M. A. JORGENSEN, Ministry of Agriculture and Fisheries. *Some results on regularity of ultrafilters.*

Notions of regularity of ultrafilters in the theory of ultrapowers were introduced largely to prove cardinality results. Usually proofs have been given to derive a property of an ultrapower from some property of the associated ultrafilter. Here we give a result in the reverse direction. In fact we prove the equivalence of an order-property of an ultrapower with a regularity property of the
associated ultrafilter. The technique of the proof has been applied by Miroslav Benda to prove that it is consistent with the axioms of set theory (ZF) that all uniform ultrafilters on a successor cardinal have the regularity property in question. An outline of Benda's proof is given. Applications of the above results to the measurement of pasture production are not discussed.

M. F. O'Reilly, University of Papua New Guinea. Extended modular characters.

We establish the existence of certain complex-valued class functions on a group $G$ which extend the notion of Brauer characters. Let $M(k)$ be the set of all similarity classes $[A]$ of square matrices $A$ with entries in a field $k$. Define addition in $M(k)$ by $[A] + [B] = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ and multiplication by $[A][B] = [A \times B]$ where $A \times B$ is a direct product. $M(k)$ is then extended in a natural manner to a commutative algebra $M_K(k)$ over a field of $K$ of zero characteristic. Theorem. If $K$ is algebraically closed and $0 \neq a \in M_K(k)$ then there exists an algebra homomorphism $\phi : M_K(k) \to K$ such that $\alpha \phi \neq 0$. Thus if $G \to GL(n,k)$ be a matrix representation of $G$ with $x \to X$, then in the notation of the theorem, choose $K = C$ and some $\phi$ and associate with $x$ the value $[X] \phi$. If char $k = p$ divides $G:1$ these class functions restricted to $p$-regular classes are just the Brauer characters. They are however defined and non-zero (though not all distinct) on all conjugacy classes of $G$.

K. L. Teo, Massey University. Modular Euclidean Lie algebras and Chevalley groups.

Our purpose is to study the structure of Euclidean Lie algebras over modular fields and to produce the Chevalley groups and Steinberg groups, in a uniform way, as groups of automorphisms on a certain type of simple quotients of the Lie algebras. The work has been done in the case when the field is of characteristic zero, except for the algebra $G_{2,3}$. Here is a résumé. Let $k$ be a field of characteristic $p$ (including 0). Let $K$ be a non-archimedean valuation field of characteristic zero, such that $k$ is the residue class field of $K$. Let $\mathcal{O}$ be the ring of integers of $K$. We construct three algebras $E_K^*, E_0^*, E_k^*$ over $K$, $\mathcal{O}$ and $k$ respectively, from a Euclidean Cartan matrix. Each of these algebras has a tier number associated with it, and this is one of 1, 2, or 3. We determine the centre of $E_K^*$. From $E_K^*$ we factor out its centre and
obtain a centre free Lie algebra $E$. We prove as in the characteristic zero case that every ideal of $E$ is principal and is of co-finite dimension. We then show that the usual "up-down property" holds in our root system and leads to a very important theorem that allows us to move around in the root spaces corresponding to a given root string. Now for each unit of $0$, we can factor each of our algebras by a certain type of maximal ideal and get finite dimension-al simple Lie algebras $E_K^0(\mu)$, $E_0^0(\mu)$ and $E(\mu)$ over $k$, $0$ and $k$ respectively. We show that $E(\mu)$ falls into the class of Chevalley algebras, and that $E(\mu)$ splits over $R(\mathcal{N}_\mu)$, where $r$ is the tier number. We also show that the properties of $E(\mu)$ can be studied by lifting to $E_K^0(\mu)$ or $E_0^0(\mu)$, so that it shares most of the properties of the characteristic zero case. By using this idea we prove $E(\mu) \cong E(\nu)$ iff $k(\mathcal{N}_\mu) = k(\mathcal{N}_\nu)$. We are able to construct Chevalley groups on each of the algebras $E_K^0$, $E_0^0$ and $E_K^0$. The last one induces a group $G_{\mu}$ of automorphisms on $E(\mu)$. Again we show that the study of this group can always be carried out by lifting to the characteristic zero case. In this way we show that if $\mathcal{N}_\mu \in k$, then $G_{\mu}$ is the standard Chevalley group, and if $\mathcal{N}_\mu \notin k$, then $k(\mathcal{N}_\mu) \oslash_k E(\mu)$ has a diagram automorphism of order $r$ and $G_{\mu}$ is the Steinberg group obtained from the adjoint group of $k(\mathcal{N}_\mu) \oslash_k E(\mu)$ by using the diagram automorphism. CONJECTURE 1: There are Weyl groups related to the $E_K^0$ and $E(\mu)$ respectively. Can we obtain the latter from the former by natural inductions? CONJECTURE 2: What is the relation between the $B-N$ pairs of the Chevalley groups on $E_0^0$ and $E(\mu)$?

REFERENCES

W. D. WALLIS, University of Newcastle, Australia. Maximal sets of one-factors.

A one-factor in a graph is a set of pairwise-disjoint edges of the graph which between them contain all the vertices. We discuss one-factors of the complete graph $K_{2r}$ on $2r$ vertices. A set $S$ of less than $2r-1$ of these one-factors is called maximal if they are pairwise edge-disjoint, and if there is no one-factor of $K_{2r}$ which is edge-disjoint from all members of $S$. **THEOREM:** A maximal set in $K_{2r}$ must contain at least $r$ one-factors; if $r$ is even then the smallest possible maximal set has $r+1$ members; both of these bounds can be realized. In fact, if $n$ is odd and $n \leq r$, then $K_{2r}$ contains a maximal set of $2r-n$ one-factors.

**ANALYSIS AND TOPOLOGY**

C. P. CHANG, University of Auckland. Divergence almost everywhere of double Rademacher series.

For positive integers $k$ and $\ell$ let $r_k$ and $r_{k\ell}$ denote Rademacher functions of one and two variables where $r_{k\ell}$ is defined by $r_{k\ell}(x,y) = r_k(x)r_\ell(y)$. J. Khintchin and A. N. Kolmogorov have shown that the condition $\sum |c_k|^2 = \infty$ is sufficient to imply divergence almost everywhere of the Rademacher series $\sum_{k} c_k r_k(x)$. The condition is also necessary because of a classical theorem which asserts that the condition $\sum |c_k|^2 < \infty$ is sufficient to imply convergence almost everywhere of $\sum_{k} c_k r_k(x)$. For double Rademacher series $\sum_{k\ell} c_{k\ell} r_{k\ell}(x,y)$ there exist examples which show that the condition $\sum |c_{k\ell}|^2 = \infty$ is not sufficient to imply divergence almost everywhere of the series $\sum_{k\ell} c_{k\ell} r_{k\ell}(x,y)$. Here divergence or convergence of a double series is interpreted by divergence or convergence of the sequence of square partial sums respectively. In this paper we obtain a sufficient condition, in terms of the coefficients $c_{k\ell}$, for divergence almost everywhere of the double Rademacher series $\sum_{k\ell} c_{k\ell} r_{k\ell}(x,y)$. The proof is based on an integral.
inequality of double Rademacher polynomials which is established by repeated application of Bessel's inequality of infinite series of orthogonal functions.

REFERENCE

D. B. GAULD, University of Auckland. *A brief introduction to catastrophe theory.*

Catastrophe theory was born about ten years ago when Thom suggested a classification of singularities of certain smooth functions. Let \( f: \mathbb{R}^n \times \mathbb{R}^r \rightarrow \mathbb{R} \) be a smooth function and let
\[
\mathcal{M}_f = \{ (x,y) \in \mathbb{R}^n \times \mathbb{R}^r \mid \frac{\partial f}{\partial x_i} = 0, i=1, \cdots ,n \} .
\]
Let \( \chi_f : \mathcal{M}_f \rightarrow \mathbb{R}^r \) be induced by projection on the second coordinate. In general we might expect \( \mathcal{M}_f \) to be a smooth \( r \)-manifold. \( \chi_f \) is called the *catastrophe map*. Let \( F \) denote the space of \( \mathcal{C}^\infty \) functions on \( \mathbb{R}^{n+r} \) with the \( \mathcal{C}^\infty \) topology. THEOREM. If \( r \leq 5 \) then \( \exists \) an open dense subset \( F_* \subset F \) so that \( \forall f \in F_* \) (such an \( f \) is called *generic*), (i) \( \mathcal{M}_f \) is an \( r \)-manifold, (ii) Any singularity of \( \chi_f \) is equivalent to one of a finite number of singularities, called *elementary catastrophes*, (iii) \( \chi_f \) is locally stable.

The number of elementary catastrophes depends only on \( r \), and for \( r = 1, 2, 3, 4, 5 \) the number of elementary catastrophes, respectively, is 1, 2, 5, 7, 11. There are many applications of catastrophe theory to such diverse areas as Biology, Economics, Linguistics, Medicine, Physics, Psychology, Sociology. Some of these applications will be described.

REFERENCES

F. B. JONES, University of Canterbury and University of California, Riverside. *Some recent results on homogeneous continua.*

A continuum \( M \) (i.e. a compact connected metric space) is *homogeneous* if for each pair \( x \) and \( y \) of its points there exists...
a homeomorphism of $M$ onto $M$ which takes $x$ to $y$. A simple example is a circle or a torus. Recently Effros has proved a theorem (in topological groups) from which it follows that given an $\varepsilon > 0$ there exists a $\delta > 0$ such that if $x$ and $y$ are in $M$ and $d(x,y) < \delta$ there is a homeomorphism of $M$ onto $M$ that takes $x$ to $y$ and moves no point more than $\varepsilon$. With the help of this tool several old problems can be solved and several proofs of known results can be greatly simplified. Ungar has recently shown that every 2-homogeneous continuum is locally connected ($M$ is 2-homogeneous if there is a homeomorphism of $M$ onto $M$ that takes a given pair of points onto any other given pair). Also I have a rather simple proof of Bing's theorem that a simple closed curve is the only homogeneous continuum that contains an arc and is embeddable in the plane. And in a completely new direction Hagopian has shown that a star-like homogeneous continuum has the fixed point property for homeomorphisms.

D. B. SAWYER, University of Otago. Uncovering.

The ideas of packing and covering are developed in [1,2]. The origins of the topics lie in the interaction between lattices in $\mathbb{R}^n$ and certain classes of sets in $\mathbb{R}^n$, notably convex bodies and star bodies. These interactions are also the central topics of the geometry of numbers. $\mathbb{Z}^n$ denotes the integer lattice in $\mathbb{R}^n$, i.e. the set of points with integer coordinates; $A + B$ denotes $\{x + y : x \in A, y \in B\}$; and $M$ denotes the set of real $n \times n$ matrices of determinant 1. Two basic theorems of the geometry of numbers are: (4) (Minkowski) If $L$ is a convex body in $\mathbb{R}^n$, symmetrical in $O$, and with volume $V(L) > 2^n$, then $\forall M \in M$, $ML \cap \mathbb{Z}^n \supset \{0\}$. (5) (Hlawka) If $L$ is a star body in $\mathbb{R}^n$, symmetrical in $O$, and with volume $V(L) < 2\zeta(n)$, then $\exists M \in M$ such that $ML \cap \mathbb{Z}^n = \{0\}$. These results can be interpreted in terms of packing: (A) is equivalent to the almost trivial remark that the density of lattice packing of congruent symmetrical convex bodies in $\mathbb{R}^n$ is at most 1; while (B), if applied to convex bodies, gives a lower bound to the best lattice packing density of congruent symmetrical convex bodies in $\mathbb{R}^n$. But (B) can be interpreted, as in the form in which it is stated above, as an uncovering result. This paper is concerned with such results. That is, let $C$ be some class of sets, let $G$ be some group acting on $C$, and let $\Gamma$ be some point set. Then we consider results of the following form: If $T \in C$ then $\exists g \in G$ such that $gT \cap \Gamma = \phi$. Several such results are described. For instance,
if \( A = \{a_1, \ldots, a_k\} \) is a set of \( k \) points on the sphere 
\[{x : |x| = 1}\] in \( \mathbb{R}^n \), and \( T \) is a set on the sphere, of measure 
\( m(T) < \frac{J}{k} \), where \( J \) is the measure of the sphere, then there 
is a rotation \( \rho \) such that \( \rho T \cap A = \emptyset \).

REFERENCES

T. O. TO, University of Canterbury. *A theorem on strongly measurable functions.*

The Denjoy Theorem ([1]; [2]; [5, p.132]) states that a real-valued function from \( \mathbb{R}^n \) Lebesgue measurable if and only if it is continuous almost everywhere relative to the density topology on \( \mathbb{R}^n \) and the usual topology on \( \mathbb{R} \). In [3], Martin proved the corresponding theorem for some category measure spaces. A theorem which generalizes the above result for vector-valued functions will be presented. Let \((X, \mathcal{A}, \mu)\) be a totally \( \sigma \)-finite measure space such that every singleton subset of \( X \) is measurable, with measure zero. Let \( \mathcal{D} \) be the density topology on \( X \) induced by the metric density with respect to a given Vitali system of measurable sets of finite measure (cf. [4], p.209; [3]). Let \( B \) be a Banach space with the metric topology \( \tau \), and let \( f : X \to B \) be a function.

**THEOREM.** \( f \) is strongly measurable if and only if \( f \) is \( \mathcal{D} \)-\( \tau \) continuous almost everywhere. A sketch of the proof will be shown.

REFERENCES

W. J. WALKER, University of Auckland. *Some applications of spectral methods to singular Cauchy problems.*

This talk outlines the spectral methods for higher order equations as developed by R.W. Carroll and applied to singular Cauchy problems. We see how these methods give existence and uniqueness theorems in an operator theoretical setting and briefly
consider additional techniques which are required to extend these results to new situations involving more independent variables.

C. S. WITHERS, Applied Mathematics Division, DSIR, Wellington. *Fredholm integral equations on an arbitrary measure space.*

Fredholm gave a solution to the integral equation

\[ f - \lambda nf = g \quad \text{where} \quad g(\cdot), N(\cdot, \cdot) \text{ are given functions on} \ [0, 1] \text{ and} \ nf(x) = \int N(x, y)f(y)dy. \]

The theory has found wide applications in the applied sciences. Fredholm's condition that \( N, g \) be bounded was weakened by Hilbert, Mikhlin, Carleman and Smithies to

\[ \| n \| < \infty, \quad \| g \| < \infty \quad \text{where these are the} \ L_2 \text{ norms of} \ N, g. \]

Smithies also extended the theory from \([0, 1]\) to \( R = (-\infty, \infty) \) and \( C \), the complex plane. Other authors about 1932 (Lichenstein, Gunther and Kneser), motivated by problems in physics, extended the measure from Lebesgue on \([0, 1]\) to a mixture of the measure giving weight to a finite number of points and Lebesgue measure on \( R^2, R^3 \). Let \( A \) be a \( \sigma \)-algebra of subsets of a set \( \Omega \) and \( \mu \) a measure on \((\Omega, A)\). We give Fredholm's theory firstly for arbitrary separable Hilbert spaces and secondly in a stronger form for the particular case

\[ H = L_2 = \{ f: (\Omega, A, \mu) \to C^\infty, \int |f|^2d\mu < \infty \} \]

with inner product \( \langle f, g \rangle = \int g^*f d\mu \) and \( n f = \int N(\cdot, y)f(y)d\mu(y) \). For \( N \) symmetric, the usual eigenfunction expansions are generalised. Applications in statistics and electrical engineering are referred to.

**REFERENCES**


A. ZULAUF, University of Waikato. *The distribution of Farey numbers.*

Let \( n \) be a positive integer, let \( u \) be a positive number, and put

\[ M(u) = \sum_{a=1}^{[u]} \mu(a), \quad \phi(u) = \sum_{a=1}^{[u]} \phi(a), \quad K = \phi(n), \]

\( \mu \) and \( \phi \) are Moebius' and Euler's functions. Let \( q_0, q_1, \ldots, q_K \)
be the Farey numbers of order \( n \), arranged in order of increasing magnitude, and put
\[
\Delta(n) = \sum_{k=1}^{K} \left( \frac{q_k}{K} - \frac{k}{K} \right)^2,
\]
\[
H(n) = \sum_{k=1}^{K} \left( \frac{Q_k^2}{K^2} \right),
\]
\[
S(u) = \sum_{\alpha=1}^{[u]} M(\frac{u}{\alpha}) \frac{1}{\alpha},
\]
\[
T(u) = \sum_{\alpha=1}^{[u]} \sum_{b=1}^{[u]} M(\frac{u}{\alpha}) M(\frac{u}{b}) \frac{(a, b)^2}{ab}.
\]

Suppose that (1) \( M(u) = O(u^\beta) \), where \( \frac{1}{2} < \beta \leq 1 \). The estimate (1) holds trivially for \( \beta = 1 \), and it will hold for every \( \beta > \frac{1}{2} \) if and only if Riemann's hypothesis is correct. It has been known for some time that (1) is equivalent to each of the following:

(2) \( \Delta(n) = O(n^{2\beta-2}) \), (3) \( H(n) = O(n^\beta) \), and obvious conclusions can be drawn about the evenness of distribution of the Farey numbers over the interval \([0, 1]\). The crucial step in the proofs which have so far appeared in the literature involves the evaluation of a certain integral in two ways, resulting in the formula

(4) \( \Delta(n) = \frac{1}{12K} \{ T(n) - 1 \} \). However, it is possible to prove (4), and the equivalence of (1), (2) and (3), by entirely arithmetical methods. The new proofs involve repeated applications of the following. \( \text{LEMMA.} \) If \( f \) is defined on \([0, 1]\), and \( 0 < \lambda \leq 1 \),

then
\[
\sum_{k=1}^{K} f(q_k) = \sum_{\alpha=1}^{n} M(\frac{n}{\alpha}) \frac{[\alpha \lambda]}{\lambda} f(\frac{x}{\alpha}).
\]

Equation (4) is obtained by considering the sum
\[
\sum_{\alpha=1}^{n} M(\frac{n}{\alpha}) \frac{1}{\alpha} \sum_{j=1}^{K} g(\alpha q_j, \lambda) \]

where
\[
g(u) = (u - [u] - \frac{1}{2})^2 - \frac{1}{12}.
\]

The remaining results are obtained by showing that (1) \( \Rightarrow \) (5) \( \Rightarrow \) (2) \( \Rightarrow \) (3) \( \Rightarrow \) (6) \( \Rightarrow \) (1), where

(5) \( T(n) = O(n^{2\beta}) \), (6) \( S(n) = O(n^\beta) \). Historical references are given in the report cited below.

REFERENCE

PROBABILITY AND STATISTICS

W.M. BOLSTAD, University of Waikato. On the distribution of the ratio of two normally distributed random variables.

This paper develops an expression for the distribution of the ratio of two independent normally distributed random variables. It shows that for the special case where $X$ and $Y$ are normally distributed with means equal zero and variances equal one, the ratio $X/Y$ has the Cauchy distribution ("student's" $t$ distribution with one degree of freedom); a well-known result. The paper then shows how these results can be extended to ratios of normally distributed random variables with correlation not equal to zero. Part two of the paper shows how this distribution can be approximated when $\frac{\mu_Y}{\sigma_Y} > 3$. This approximation enables the distribution to be found from standard normal tables. Then the paper shows how the distributions of non-central chi square, non-central $t$, and non-central $F$ random variables can be found from the standard normal tables using these approximations. These could be used to establish the power functions of many tests used in the analysis of variance.


This is a report on some work carried out by Bruce Hutton and myself. Consider the usual linear regression model $Y = X \beta + \epsilon$ where $Y$ is a vector of $n$ observations, $X$ a known $n \times m$ matrix, $\beta$ a vector of $m$ parameters to be estimated, and $\epsilon$ a vector of $n$ independent random variables with mean, 0, and variance $\sigma^2$. It may happen, in practice, that $X$ cannot be measured exactly, rather $\tilde{X} = X + \Delta$ is known, where $\Delta$ is the measurement error. Then the usual least square estimates for $\beta$ and $\sigma^2$ with $\tilde{X}$ in place of $X$, namely, $\hat{b} = (\tilde{X}'\tilde{X})^{-1} \tilde{X}'Y$, $\hat{\sigma^2} = Y'[(I - \tilde{X}(\tilde{X}'\tilde{X})^{-1} \tilde{X}')Y/(n-m)$, will be subject to bias. This paper is concerned with determining the magnitude of this bias and, in particular, with finding simple criteria for deciding whether it is likely to be serious. It will be supposed that the orders of magnitude of the elements in each of the columns of $\Delta$ are known. These will be denoted by $r_1, \ldots, r_m (\geq 0)$ and it will be convenient to let $R = \text{diag}(r_1, \ldots, r_m)$. The problem will be considered from three points of view: (i) A 'distance from singularity' of the matrix, $\tilde{X}'$, will be defined and it will be suggested that if this is too small at least some components of $\hat{b}$ will be meaningless. (ii) It will be supposed that $r_1, \ldots, r_m$ are, in fact, the r.m.s. values of the columns of $\Delta$, and a bound is found on the maximum bias introduced by $\Delta$ into any component of $\hat{b}$. In particular, an easily calculated criterion is found for deciding whether this error can be significant in comparison with the standard deviations of the components of $\hat{b}$. 

92
(iii) The elements of $\Delta$ are regarded as random variables with the rows of $\Delta$ being independent and identically distributed, the elements in the $i$-th column having variance $\sigma_i^2$. Asymptotic formulae for the bias introduced by $\Delta$ (for large $n$) are found and again a simple criterion for assessing the significance of the error is given.

REFERENCES

F.C. DURLING, University of Waikato. *Bivariate normit, logit and burrit analysis.*

The problem of a mixture of two drugs in a biological quantal assay is investigated empirically making the basic assumption that the form of the response region remains constant irrespective of the biological considerations and that the probability functions themselves may take on different forms dependent on these biological considerations. The bivariate normal distribution, a bivariate logistic distribution, and a bivariate Burr distribution are utilized in this investigation. The analyses of data include examples which had been classified by previous investigators as synergistic action, simple similar action, independent action and additive action. Fortran IV programs are included which will illustrate the functional forms utilized in the investigation.


A bivariate renewal process is a sequence of independent and identically distributed non-negative bivariate random variables $\{(X_n, Y_n)\}, (n=1,2,\ldots)$. If $(S_n^{(1)}, S_n^{(2)}) = \left( \sum_{i=1}^{n} X_i, \sum_{i=1}^{n} Y_i \right)$, define

$$N_x^{(1)} = \max \{ n : S_n^{(1)} \leq x \}, \quad N_y^{(2)} = \max \{ n : S_n^{(2)} \leq y \}, \quad N_{x,y} = \min(N_x^{(1)}, N_y^{(2)}).$$

$N_x^{(1)}$ and $N_y^{(2)}$ are each univariate renewal counting processes. In two papers ([1], [2]) published recently a general study was made of the bivariate renewal counting process $(N_x^{(1)}, N_y^{(2)})$ and the two dimensional renewal
counting process \(N_{x,y}\). In this paper we elaborate on the results concerning the asymptotic properties of the distribution and moments of both \((N_x^{(1)}, N_y^{(2)})\) and \(N_{x,y}\). In addition a discussion on research at present being carried out on some related problems is presented.

REFERENCES

G.C. Jain, Otago University. Busy period distributions and their applications.

This paper reviews probability distributions of the number of customers served in the busy period of single server queues with Poisson input and different service times. The results are extended to the multivariate case in which different types of customer arrive each according to a Poisson law. Applications to traffic flows and spatial distributions are discussed. The relations of these busy period distributions to such areas as random walks, first passage problems and branching processes are described.


If the distribution of a variate \(X\) in a population is known, the mean value of the dependent variate \(Z\) is given by the regression formula \(Z = \theta(X) + E\) (where \(E\) is random with variance \(\sigma^2_E\)) and may be estimated without bias and with improved accuracy by the mean of a sample representative in its distribution of \(X\) rather than a strictly random sample. This procedure, which does not require knowledge of \(\theta(X)\), is exploited in two-phase sampling, where a large sample of values of \(X\) supplies a sampling frame for a much smaller sample of values of \(Z\). For very small second phase samples (e.g. \(n = 2\)) strict representativeness cannot usually be secured, but with certain limitations on the form of the regression the same ends may be achieved. Thus, for example, if \(X_1\) and \(X_2\) are taken at \(\mu_X \pm \sigma_X\) and \(\theta(X)\) can be assumed to be of the form \(\theta(X) = \alpha + \beta X + \gamma X^2\) then \(\frac{1}{x}(\theta(X_1) + \theta(X_2)) = \alpha + \beta \mu_X + \gamma (\mu_X + \sigma_X^2) = E(\theta(X))\) and therefore, \(E(\frac{1}{x}(Z_1 + Z_2)) = \mu_Z, \var(\frac{1}{x}(Z_1 + Z_2)) = \frac{1}{x^2}\sigma^2_E\). Knowledge of the values of \(\alpha, \beta, \gamma\) is not required. Applications may be found in pasture sampling, where the direct measurement of herbage dry matter requires expensively tedious and
destructive cuts, whereas height (which varies in its relationship with dry matter according to the condition of the herbage) may be measured quickly and easily. This technique, which has features in common with Gaussian quadrature, may be generalised to more than one first-phase variate and a more general form of regression relationship.


A stationary second order process \( \{x(t), t \in \mathbb{R}\} \) is "band-limited to \( W \)" if its covariance function \( R(t) \) has the representation

\[
R(t) = \int_{-\infty}^{\infty} e^{2\pi it\lambda} \mu(d\lambda) \text{ where } \mu \text{ is a positive finite measure with support in } [-W, W].
\]

Such random processes satisfy the "sampling theorem" \( x(t) = \sum_{n=-\infty}^{\infty} x(nh) \frac{\sin \pi h^{-1}(t-nh)}{\pi h^{-1}(t-nh)} \) for all \( h^{-1}/2 > W \). This result permits the reconstruction of a signal from a knowledge of the amplitudes at the "sampling points" \( nh, n = 0, \pm 1, \pm 2, \ldots \). In this paper we discuss generalisations of this theorem and present sampling theorems satisfied by more general classes of non-stationary processes.

B.F.J. MANLY, University of Otago. The double exponential fitness function.

A fitness function is a function that relates the probability of an individual surviving natural selection to quantitative and qualitative characteristics of the individual. A double exponential fitness function is of the form

\[
w(X_1, X_2, \ldots, X_p) = \exp\{-\exp(\alpha_0 + \alpha_1 X_1 + \ldots + \alpha_p X_p)\}, \tag{1}
\]

where the \( \alpha_i \) are constants and the \( X_i \) are variables associated with individuals. In this paper the double exponential fitness function is compared with other types of fitness function [1,3], and is shown to be the most reasonable in some ways. A relatively simple empirically weighted least squares method for fitting the double exponential to data is then discussed and justified by some sampling experiments. The paper ends with an application to Karn and Penrose's [2] data on the survival of human infants related to sex, birth weight and gestation time. An analysis of these data suggests that (i) the optimum birth weight is about 8 lbs for all gestation times, (ii) the optimum gestation time is about 288 days for all birth weights, and (iii) boys are about 13% more susceptible to death than girls for all birth weights and gestation times.

REFERENCES


In this paper, the efficiencies of the Kruskal-Wallis and C-sample normal scores tests are studied in the case of (1) the Poisson distribution, and (2) data, grouped (to the nearest integer, or nearest first decimal etc.) from the normal, exponential, logistic, double exponential, Cauchy and rectangular distributions.


Randomisation typically arises when applying tests of significance to discrete random variables. Arbitrarily chosen significance levels cannot in general be achieved without resorting to an auxiliary random experiment that is independent of the data. Although randomisation is required in the derivation of certain optimal tests (see Lehmann [1]), many authors have expressed opposition to its use: "... randomisation is seldom if ever used in practice, mainly for two reasons. First, we are interested more in the level of significance than whether or not a particular value is exceeded; and secondly, inferences should not depend on a device which bears no relation to the problem studied." [2, p.9] Starmer, Grizzle and Sen [3, p. 377] seem prepared to compromise: "Even though most statisticians would not use the randomised test in practice, it could be used for judging the value of competing tests. Thus we could search for the best approximation to the most powerful test that does not require the undesirable feature of randomisation to achieve the desired significance level." Arguments for and against randomisation are raised, and in the discussion time it is hoped that members will declare their preference.

REFERENCES
R. L. BROUGHTON, University of Canterbury. The solution of systems of nonlinear algebraic equations.

The purpose of this paper is to review the work of the numerical analysis group under Professor C.G. Broyden at the University of Essex. If \( f(x) = 0 \) represents a system of \( n \) nonlinear algebraic equations in \( n \) unknowns, then Newton's method may be written

\[
J(x_k)s_k = -f_k,
\]

\[
x_{k+1} = x_k + t_k s_k, \quad (t_k \text{ a scalar}).
\]

In order to overcome difficulties arising from the use of the Jacobian matrix, \( J \), of \( f \), Broyden [2] proposed approximating \( J \) and updating this approximation by a matrix modification method. Another class of methods of solution of \( f(x) = 0 \) is the continuation methods, [3]. Here the main problem is embedded in a family of problems. A sequence of intermediate problems may then be generated so as to yield the solution to the main problem. A variation on this approach has been to differentiate the equations obtained from the imbedding process. A system of ordinary differential equations is generated and these equations may then be solved by numerical techniques for differential equations, [1].

REFERENCES


R. D. BROWNRIGG, Victoria University of Wellington. The application of variable metric minimisation techniques to the N-stage decision problem of dynamic programming.

The use of variable metric minimisation techniques in solving the N-stage decision problem of dynamic programming promises to make savings in both computation time and high-speed storage, at the same time alleviating the well known 'curse of dimensionality' of dynamic programming. The algorithm proposed, which is essentially a second-order gradient method, has the property of finite convergence for the problem with linear constraints and quadratic criteria (the L.Q.P. problem), and involves the generation of a quadratic approximation to the cost function as a function of the state vector at each stage. Variable metric minimisation techniques are used to generate the information necessary to make this quadratic approximation to the cost function in the region of a nominal (non-optimal) trajectory,
and this information is then used to update the trajectory in such a way that an overall reduction in the cost function is achieved. A similar second order gradient method, known as differential dynamic programming, has already been proposed by Jacobson and Mayne, [1,2], and the differences between the two methods are outlined. Possible variations of the basic algorithm are also outlined.

REFERENCES

D. N. FOSTER, University of Otago. *Towards a methodology for producing robust solutions using data with high inherent uncertainty.*

Very many operations research projects use deterministic techniques, such as mathematical programming. However there are often many potential sources of error in the data, and these deterministic techniques often have no built-in means of coping with this error. Because these techniques search for a solution at an extreme point of the solution space, this raises doubts about the practical validity of so-called optimum solutions. The standard answer to this problem is sensitivity analysis, and in the case of mathematical programming, the whole field of stochastic programming (in particular chance constrained programming) has been developed to deal with the problem. This paper describes a similar approach applied to a fuel supply problem which could not be solved by these methods. It concerned the scheduling of supplies of (blended) low grade fuel from a number of discrete sources to a thermal power station over a period of years. Specifications for the fuel (the most important relating to three major parameters of quality) were laid down by a contract which had been signed by the suppliers and the electricity authority. Fuel from several sources was below specification; hence the need for blending. Furthermore the standard errors of all the quality parameter estimates were quite large. There was thus an unacceptable risk that blends which met the specification as per the estimated quality data might in fact be of substandard quality. Time constraints did not permit the obtaining of more accurate data, so attention was directed to finding a solution that was a "safe" distance inside the solution space. First the average (over all ponds) was calculated for each of the parameters. In each case this average was found to be well inside the specification. This suggested the approach was feasible. Essentially the problem was solved by first listing all possible blends which were close to the overall average quality. These were not independent, but it was a relatively simple matter then to schedule blends from this list so as to minimise
costs while meeting other short term limitations.

REFERENCE

A. W. MCINNES, University of Canterbury. *On the convergence of numerical quadrature formulas for improper integrals.*

Davis and Rabinowitz [1] investigated the soundness of the process of "ignoring the singularity" in numerical quadrature. The primary result was that if the singularity occurs at a rational point and if the integrand is monotonic in a neighbourhood of the singularity, then "ignoring the singularity" is a theoretically valid process for a large class of sequences of composite rules, i.e. convergence to the exact value of the integral is guaranteed. Various extensions and generalizations to the basic positive result are discussed by Rabinowitz [4], Miller [3] and El-Tom [2]. The purpose of this paper is to examine a negative result in [1]: that if the singularity occurs at an irrational point then the process of ignoring the singularity may not be theoretically valid. By considering the analysis of the error of approximate integration in a functional analysis framework it is shown that this result may be refined at a theoretical level.

REFERENCES

A. J. PAYNE, University of Waikato. *Design of distributed computer systems.*

Very little of graph theory or operational research techniques has been applied to the design of distributed computer systems. The paper gives four normally unconnected techniques which provide the basis of positioning the computers of a distributed network in the most economic and suitable positions. (1) *Graph theory's connectivity matrices:* The connectivity matrices for a network allow the network to be checked for valid logic and later for partitioning into simpler systems. The flow matrices give estimates of the steady state flow in the systems so as to ensure the flows do not saturate the channels connecting the computers. (2) *Tree analysis of functions:* The dependency of system functions on each other can be represented as a tree of functions connected by 'and' or 'or' operators. The tree
can be reduced to a set of 'or'-ed subtrees which consist of 'and'-ed functions. A comparison of the functions will give a basic set of functions for each subtree to ensure that there are no duplicate functions. This will lead to a selection of the simplest set of functions to perform the work of the system. (3) Job shop scheduling: Once the basic sets of functions have been selected, they can be represented in terms of their uses of system facilities by a network (quantity against time) with logical dependences shown as dummies. We evaluate the cost of different network choices to select the design with least costs. (4) Transportation technique: The transportation technique gives a method by which the cost of the communication net is minimised. When regarded as information flow, the sources of input and supplies of output can be moved so as to give an optimal network flow and an optimal placement of sources and sinks for the cost/effectiveness of the system. The techniques described are being used to define the design of the distributed computer system being developed at Waikato University.

D. M. RYAN, University of Auckland. Scheduling via linear programming.

The formulation of linear mathematical models arising from real-world applications of scheduling or routing is illustrated, and the computational complexity of such models is discussed. It is seen that this complexity prohibits the use of standard L.P. packages as solution procedures, but in certain circumstances the application of the L.P. column generation technique overcomes the complexity problems and provides a powerful tool for the optimal solution of the models.

J. A. SHANKS, University of Otago. Error expansions for Romberg integration.

One of the basic problems in numerical analysis is the approximation of the value of the integral $I = \int_R f(x) \, dx$ (assumed to exist) by a calculable value $T(h)$ depending on some parameter $h > 0$. In the case of a one-dimensional integral with $R = [a,b]$ then $T(h)$ could be the value of the trapezoidal rule applied with step-length $h$ (assumed to be $\frac{b-a}{n}$ for some $n \in \mathbb{N}$). If we can find an (asymptotic) expansion of the form (1) $T(h) = I + a_1 h^{n_1} + a_2 h^{n_2} + a_3 h^{n_3} + \cdots$, where $n_1 < n_2 < n_3 < \cdots$, then the method of Romberg Integration (an example of Richardson's "extrapolation to the limit") enables us systematically to improve our approximation by essentially eliminating successive error terms in (1). This can be achieved by forming suitable combinations of values of $T(h)$ for different values of $h$;
for example \( \frac{2^n T(h) - T(\frac{h}{2})}{2^n - 1} = I + b_2 h^{n2} + b_3 h^{n3} + \cdots \), for some \( b_2, b_3, \cdots \), and the combination on the left can generally be expected to have smaller error than \( T(\frac{h}{2}) \). Such expansions as (1) are known or easily derived for one-dimensional integrals with or without various types of end-point singularities in the integrand, for composite versions of all the common quadrature rules (Newton-Cotes, Gaussian, etc.). The present paper discusses the analysis of various two-dimensional singular integrals for different approximating rules \( T(h) \) (composite product and non-product rules are considered) in order to derive expansions of the form (1) or similar. The approach is to reduce a given rule into a linear combination of basic rules, for which the analysis is simplified. The treatment allows for both point and line singularities in the integrand (assumed to lie on the boundary of the rectangular region \( R \)).

G. J. TEE, University of Auckland. *An ALGOL 60.1 compiler in ALGOL 60.1.*

"An ALGOL 60 Compiler in ALGOL 60" gives an ALGOL 60 program which translates an ALGOL 60 program into a sequence of macro-instructions, for a hypothetical stack machine with a repertoire of cl40 instructions. Each such macro-instruction is then further translated into assembler (or machine) code for a specified machine. The published program is oriented towards the EL-X8 computer, with 27-bit words and Flexowriter input (with underlining). It has largely been rewritten to operate on any machine which will represent all integers with modulus less than \( 2^{15} \) (e.g. on any binary machine with 16 or more bits per word). The source program can use any character codes, with reserved words instead of underlining. The target machine is specified by supplying the machine code for each type of macro-instruction of the hypothetical machine. (These could be subroutine calls). The compiler has been implemented initially on the B6700 computer, translating ALGOL programs into code for the ALPHA LSI-2 computer. It is intended to use the compiler to compile itself into ALPHA code. The compiler is designed to compile source programs written in ALGOL 60.1, which is a revision of ALGOL 60 proposed [2] by IFIP WG/2.1.

REFERENCES


P. J. BRYANT, University of Canterbury.  Water wave stability.

The first approximation to water wave motion is the linear theory of dispersive infinitesimal waves. Higher approximations for waves of small but finite amplitude describe nonlinear interaction effects which are weak, but which may cause slow changes in amplitude and phase velocity. In the linear theory, any sinusoidal wave propagates over water of uniform depth with a constant velocity without change of form. When nonlinear effects are included, it is only the permanent waves, which include Stokes waves and cnoidal waves, that propagate with a constant velocity without change of form. Stokes waves of almost all wavelengths have been shown by Benjamin and others to be unstable to sideband disturbances. Cnoidal waves, which are permanent waves on shallow water, are stable to most but not all disturbances. Even when a cnoidal wave is stable, the margin of stability can be made to be so small that small disturbances cause large changes to the wave properties. It is found that the large changes are periodic, indicating that in such cases cnoidal waves are stable in a nonlinear sense.

W. DAVIDSON, University of Otago.  Aspects of Newtonian Cosmology.

A review of the basic ingredients of Newtonian cosmology is presented. The status of the inertial or Newtonian reference frame, the status of spin and shear in such frames, the essential indeterminancy of the equations of motion and the question of a fundamental singularity are investigated in the light of recent researches by Davidson and Evans and other authors.

W. DAVIDSON and H.J. EFINGER, University of Otago.  On the measure of proper time in general relativity.

According to general relativity, the proper time between two events on a time-like path is the aggregate of infinitesimal increments of time as measured by standard clocks in consecutive inertial frames along the route. In view of this can the lapse of proper time for an accelerating rocket be measured satisfactorily by an atomic clock on board? Or can the Einstein redshift in stellar atomic line emissions be trusted to be independent of the fact that the matter is accelerating relative to local inertial frames? What about nuclear clocks used to measure gravitational redshifts; what about the decay times of accelerated fundamental particles? An investigation of the order of magnitude of likely effects is made.
A. B. EVANS, University of Otago. Correlation of Newtonian and relativistic cosmology.

The usual idealized Newtonian universe is an infinite region of pressureless "dust", of uniform density $\rho(t)$, with a streaming motion obeying a linear law: $\mathcal{V} = H \mathbf{x}$, $H = H(t)$. $H$ can be split into an expansion scalar $\dot{H}$, a traceless symmetric shear tensor $\mathbf{Q}$, and an antisymmetric spin tensor $\mathbf{W}$: $H = \dot{H} + \mathbf{Q} + \mathbf{W}$. A particle initially at $\mathbf{x}_0$ has the general position $\mathbf{x} = A \mathbf{x}_0$, where $A = A(t)$, $\dot{A} = HA$, $A_0 = I$. The gravitational potential has the form $U = \frac{1}{2} \rho \dot{x}^i \dot{x}^j$, where $\rho = \rho(t) = \rho^t$. Euler equation: $\dot{H} + H^2 + \rho = 0$. Poisson equation: $\text{tr}(\rho) = 4\pi G \rho$ ($\rho$ indeterminate). Continuity equation: $\dot{\rho} + \rho \text{tr}(H) = 0$. The general comoving metric for a dust universe is $ds^2 = d\tau^2 + 2g_{ij}d\xi^i d\tau + g_{ij}d\xi^i d\xi^j$, with the conservation equations $\partial (\rho \sqrt{-g}) / \partial \tau = \partial g_{ij} / \partial \tau = 0$. It turns out that one can construct spatial power series expansions for the $g_{ij}$ and $g_{4i}$, so that the lowest local approximation to the comoving metric is $ds^2 = d\tau^2 + 2F_{ij}d\xi^i d\tau + M_{ij}d\xi^i d\xi^j$, where $F = -A^t \mathbf{W}A$ is constant, and $M = -A^t A$. At this level, $\tau$ and the $\xi^i$ can be identified with the Newtonian $t$ and $x^i_0$. Applying the power series approach to the relativistic field equations, we can resolve the indeterminacy of $\rho$, obtaining $\rho = A^t \mathbf{G} \rho A + \mathbf{Q} - (Q^2 - \frac{1}{3} \text{tr}(Q^2)I) + \varphi_c$, where $\varphi_c$ is related to the local spatial curvature. Besides having some heuristic value, this result helps to classify various types of singularity ($\rho \to \infty$) and expansion ($\rho \to 0$).

REFERENCES

J. D. LOVE, Australian National University. Boundary value problems involving non-orthogonal functions.

Techniques are presented for solving difference equations that arise in boundary value problems involving a class of non-orthogonal functions on the coordinate surfaces. Specific problems are solved explicitly, including the magnetohydrodynamic modes of a toroidal
plasma, the van der Waals force between two spheres and the acoustic scattering from ring transducers.

J. D. LOVE, Australian National University. *Light transmission along optical communication fibres and the phenomenon of electromagnetic tunnelling.*

Some interesting properties of the transmission of light rays through very long optical fibres used for communications purposes are presented. The attenuation of an electromagnetic mode propagating along such a fibre, or waveguide, is given a simple interpretation in terms of light rays obeying a generalization of the classical laws of Snell and Fresnel. This extension to the laws of reflection and transmission includes the effects of curvature, and exhibits the phenomenon of electromagnetic tunnelling. Absorption effects can also be included in this model.

A. D. SNEYD, University of Waikato. *Potential flow into small gaps.*

A steady current is passed between two electrodes placed on the surface of a sphere of radius $a$ and uniform electrical conductivity $\sigma$. The aim of this paper is to find an asymptotic formula for the electrical resistance between the electrodes in the limit as $L/a \to 0$, $L$ being a typical linear dimension of the electrodes. The method used is matched asymptotic expansions. We suppose that the electrodes are ellipses with major axes $a_1, a_2$ and eccentricities $e_1, e_2$. An "outer expansion" for the electric potential $V$ valid everywhere except in the vicinity of the electrodes is constructed in terms of the Neumann function for the sphere. An "inner expansion" for $V$ valid near the electrodes is derived using ellipsoidal coordinates, and the two expansions are matched. It is found that the electrical resistance is

$$\frac{1}{2\pi a_1^{\sigma}} [K(e_1) + \frac{1}{\gamma} K(e_2) - \epsilon \log \epsilon + O(\epsilon)],$$

where $\epsilon = a_1/a$, $\gamma = a_2/a_1$ and $K(m) = \int_{0}^{\pi/2} (1 - m \sin^2 \theta)^{-1/2} d\theta$, is the complete elliptic integral of the first kind. Applications of this technique to other potential flows into small gaps, and some unsolved problems are discussed.


Let $L$ be a real self-adjoint, second-order uniformly elliptic
differential operator, and consider, in a bounded region
\( \Omega \subset \mathbb{R}^n \) \((n \geq 2)\), the problem (1):
\[ L(u) = \lambda f(x,u), \quad x \in \Omega, \]
with a linear mixed boundary condition, on the boundary \( \partial \Omega \), given by
(2):
\[ B(u) = 0, \quad x \in \partial \Omega. \]
Equations (1) and (2) constitute a nonlinear eigenvalue problem with eigen-parameter \( \lambda \). The nonlinearity is caused by the function \( f \). The spectrum of the problem is the set \( \Lambda \) of \( \lambda \) for which (1) and (2) possess a positive solution. Interest here is focussed on the value \( \lambda^* = \sup \Lambda \), which is called the critical parameter of the problem, being related to the instability of the corresponding time-dependent equation. Two new properties of \( \lambda^* \) are derived. Firstly it is shown that \( \lambda^* \) is finite for a wide class of functions \( f \) and secondly that, for a different but also wide class of functions \( f \), \( \lambda^* \notin \Lambda \). These results extend, and depend on, the corresponding results in Amann [1, 2] and Keller and Cohen [3].

REFERENCES


D. F. WALLS, University of Waikato. Stochastic models of phase transitions in chemical reactions.

The far from equilibrium steady state of simple nonlinear chemical systems is analysed. Models are chosen for which a macroscopic analysis shows that the non-linearity introduces instabilities which cause transitions analogous to thermodynamic first and second order phase transitions. Fluctuations are introduced into these models by deriving a Markoffian stochastic master equation for \( P(x,t) \), the probability of having \( x \) molecules of a chemical species \( X \) at time \( t \). The steady state solutions to these master equations are obtained invoking the principle of detailed balance. In the case of the second order phase transition, the steady state probability function may be used to calculate the entropy above and below threshold. It is revealed that there is a relative decrease in entropy above the critical point revealing that the system has been driven into a more ordered state, [1]. For the first order phase transition, a macroscopic analysis reveals
that the system undergoes hysteresis between two alternative steady states. The solution for the steady state probability function is bimodal in the region of this hysteresis. Thus a quasi-hysteresis occurs involving transitions between alternative metastable steady states on a time scale that is much longer than that of the fluctuations around the mean steady state values. An estimate of the time-scale of these fluctuations is obtained by examining the corresponding Fokker Planck equation, [2].


N. A. WATSON, University of Canterbury. The Dirichlet problem for the heat equation.

The classical form of the Dirichlet problem for the heat equation
\[
\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}
\]
on a rectangle \( R = \{(x,t) : a < x < b, c < t < d\} = R_0 \times [c,d]\) in the \((x,t)\)-plane is as follows. Given a continuous function \( f \) defined on \( \delta R = (\{a\} \times ]c,d[) \cup ([a,b[ \times \{c\}) \cup ([b] \times ]c,d[)\), find a continuous function \( u \) on \( R \cup \delta R \) such that \( u \) is a solution of the heat equation on \( R \) and \( u = f \) on \( \delta R \). That a uniformly continuous \( u \) exists is a classical result. The form of the extension of this problem to an arbitrary open set \( E \) is not obvious. Recent work in axiomatic potential theory suggests that the given continuous function \( f \) should be defined on the whole of the boundary of \( E \), and that, for the rectangle \( R \), those boundary points in \( ]a,b[ \times \{d\} \) should be regarded merely as irregular points. We present and discuss an alternative form of the problem for \( E \). This is an extension of the classical form for \( R \), and gives a kind of regularity to certain points which are considered as irregular by users of the other form. The discussion carries over to \( n \)-dimensional space.

B. A. WOODS, University of Canterbury. The Hayes-Probstein solution for thin-shock-layer flow over a delta wing, with attached shock wave.

In their monograph on hypersonic flow [1], Hayes and Probstein gave a conjectural description of the solution (in the thin-shock-layer approximation) for the hypersonic flow over a delta wing, with attached shock waves. Their discussion did not however provide a prescription for the calculation of the solution, and no such calculation has meanwhile appeared. (The solution to the same problem, and in the same approximation, published by Woods in 1970 [2] differs
markedly from the H.-P. solution.) Using the general solution for
the thin-shock-layer equations developed by McIntosh and Woods [3],
a numerical procedure is given for the realization of the H.-P.
solution. Many of the features conjectured by Hayes and Probstein
do indeed appear in this, and in addition, some interesting features
not predicted by them are uncovered. Calculations using the procedure
are compared with experimental results, those obtained by the
earlier thin-shock-layer solution [2], and with results obtained by
finite difference methods. The verbal presentation ends with a
promise by the author not to write any more papers using this
approximation.

REFERENCES
2. B. A. Woods, *Hypersonic flow with attached shock waves over

MATHEMATICS EDUCATION


The idea of open book testing of mathematics has a certain appeal.
In these examinations students are allowed to refer to textbooks or
notes if they wish. The emphasis is on problem solving rather than
recall of memorised material. It seems that this testing situation
more closely approximates the 'real world' use of mathematics than is
the case with the traditional 'closed book' test. But a number of
questions arise. Do some students consistently perform better at one
sort of test than at the other? If so, what sort of students? How
does open book testing affect the learning process? Reference is made
to available research in this area and experience with this form of
testing at Massey is discussed.

D. C. JOYCE and A. SWIFT, Massey University. *Numerical analysis and
computing in the seventh form.*

In 1974 "Numerical Analysis and Computing" made its debut in the
seventh form applied mathematics syllabus. A surprisingly high
proportion of students chose to attempt questions from this section of
the Bursary and Scholarship examination papers, which indicates that
many schools have given a high priority to this new subject. In this
paper discussion is centred on the strengths and weaknesses of the
prescription, the problems it generates for the teacher and some
possible teaching approaches. Reference is also made to the effects
on university courses.
The question whether cook-books are really to be despised was put to the Committee of Professors, whose view was that it depended on the cook, and gave R. A. Fisher's Statistical methods for research workers as an example..." [1]. Elementary statistics courses which minister to the mindless use or misuse of a few standard formulae have sometimes been labelled "cookbook", often in a context where it is implied that attention to the mathematical derivations is what is needed to put the matter right. Alas, mathematical derivations are concerned with logical connections, and it is practical experience and intuition, rather than mathematical logic as such, which relates a mathematical theory to its real-world application. Perhaps the most important need is that elementary teaching should give attention to the practical investigations which should precede any formal statistical analysis. This will include the use of appropriate graphs, and the probing of assumptions which are critical to the use of the formulae. The questions at issue are illustrated with examples from one-way analysis of variance, and from the use of chi-square tests in connection with contingency tables. These points are especially relevant to the teaching of statistics at the school level. It is the practical relevance of the subject which more than anything else justifies the teaching of statistics as a school subject. Furthermore, account must be taken of the way in which statistical methods are finding application in other areas of the school curriculum, notably in seventh form biology. Mathematical derivations must not loom too large. At the school level the primary consideration in deciding whether to include a derivation should be the light which it may shed on the result and on the assumptions which lie behind it. In elementary probability the mathematical derivations are intimately bound up with the results obtained and can readily be taught in an illuminating way. The case is by no means so clear once one moves beyond probability to the inferential arguments of statistics. Suggestions are made for amending the present seventh form statistics syllabus in ways which may make it more manageable, give more attention to elementary practical details, and relate it better to other parts of the school curriculum.

REFERENCE


B. W. WERRY, Christchurch Teachers College (Secondary Division). Teacher training and the universities.
and consequently university courses taken by prospective teachers are not commonly designed with their future needs as teachers in mind. There are obvious gains to be made from a closer liaison between universities and teachers colleges, but any move towards complete integration should be approached with caution lest the advantages of the present structure be lost. These viewpoints are discussed with particular reference to pre-service training courses for secondary school mathematics teachers at Christchurch Teachers College. This country's pattern of teacher training is also compared, on the basis of observations made in the third term of 1974, with those typical of the United States, where teacher training is wholly university based.