
This book is a valuable addition to the rather short list of satisfactory textbooks dealing with electromagnetic theory at the undergraduate level. Although designed to cover the two or three years of an undergraduate course only, it compresses a remarkable amount of information into its 279 pages. The treatment is in a light but fully explanatory style which makes the book rather novel in its class. One is continually refreshed by the nuances of the author's approach to the derivation of the well established results of electro-magnetic theory. Information almost casually introduced on every page reveals that we are in the hands of an expert with a deep understanding of his subject and an impressive physical insight.

Dr. Clemmow's general approach to the subject is one that will have special appeal to the applied mathematician or mathematical physicist. Noting the ever-widening scope of applied mathematics, and the range of mathematical physics, with the resulting competitive claims on the undergraduate syllabus, the author sets out to present the vital elements of electromagnetic theory in a more selective and circumspect manner than has been conventional hitherto.

Assuming a fair mathematical competence on the part of the reader, especially in real and vector analysis (some of the major theorems of the latter are given in an appendix), the author takes initially the shortest possible route to Maxwell's equations, relying on the basic force law regarded as 'observational'

\[ \mathbf{F} = e \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \]

supported by the field laws
The relativistic modifications of the latter are ignored until more sophisticated considerations are introduced later. By a combination of logic and plausibility arguments these laws serve to lead the reader through electrostatics, Ampère's circuital law, the Biot-Savart law, Faraday's law of induction to the final generalisation of Maxwell's equations by the end of Chapter 2. In the subsequent chapters the essential rigorous consequences of Maxwell's equations are presented, with reference back to Chapters 1 and 2 as necessary. A full discussion of the electrostatic field with a phenomenological treatment of dielectrics is given in Chapter 3, the magnetic field of steady and slowly varying currents, circuit theory and magnetic media in Chapter 4. Electromagnetic waves, dipole radiation and the radiation field receive comprehensive exposition in Chapter 5. Finally in Chapter 6 a microscopic treatment of media (short of quantum considerations) deals with polarisation and magnetisation: microscopic particle models are given for the free electron gas, plasmas, dielectrics and magnetic media.

At all times, at least after Chapter 2, a concern is shown for mathematical rigour, without pedantry. The more specialised apparatus of mathematical techniques, use of Bessel functions, Legendre polynomials, tensor calculus is eschewed in favour of providing an understanding of the essentials. There is a notable concern to set the theory in sound physical perspective, without getting bogged down in technical details. Using rationalised m.k.s., or S.I. units, the author is at pains to indicate how each theoretical quantity would be measured physically as he introduces it, as well as indicating what orders of magnitude are relevant to

\[
\mathbf{E} = \frac{e}{4\pi\varepsilon_0} \frac{\mathbf{r}}{r^3} \\
\mathbf{B} = \frac{\mu_0\mathbf{E}}{4\pi} \frac{\mathbf{r} \wedge \mathbf{r}}{r^3}
\]
laboratory experience.

A total of 180 problems for solution are provided at the ends of the chapters. Many of these are far from easy and would require the guidance of the lecturer; but they are highly relevant and instructive.

The language of the text is on the whole of a high standard. An idiosyncrasy noticed was a partiality to commencing a sentence with the word "That". The Cambridge University Press have provided a good standard of printing; curiously the only misprints noticed were in the Preface (two mis-spellings, a misplaced comma, and a glaring grammatical error).

In summary Dr. Clemmow is to be congratulated on the production of a very attractive new book on electromagnetic theory, worth every new-penny of its price - £5.40 (hard cover), £2.40 (paperback).

W. Davidson


This book has many likeable features. It aims "to present an interesting approach to mathematics that does not involve complicated algebraic manipulations", and thus to encourage students who perhaps have never had a course in high school mathematics "to become involved in mathematics so that they can learn to appreciate, understand, use and enjoy it". Provided that a sympathetic tutor is at hand to help the students along, I think the book will accomplish this aim. Each new topic is introduced in a manner calculated to awaken and sustain the reader's interest. The material presented is within easy reach of the novice, and yet it reveals the essence of mathematics and affords a panoramic view of the subject. The chapter
headings are: Mathematical recreations, what is mathematics, logic, sets and paradoxes, geometry, counting and probability, statistics, linear algebra, game theory, calculus, computers, and an appendix on the ordered field of real numbers.

The book would be suitable as a text for a 'general studies' or 'mathematics appreciation' course. The teacher of such a course would find the selection and sequential arrangement of the material and the numerous examples and exercises a useful guide. However, there are several points of detail where improvement or change is desirable or necessary. For instance, it is strange that no satisfactory explanation is given of the concept and use of variables. Indeed, there is sometimes unhappy confusion between variables and the objects over which they range. Another strange lapse is that the expression $y = f(x)$ is introduced as being merely a shorthand notation for "$y$ depends on $x$". Later attempts to reveal the true significance of the symbol $f$ cannot wholly undo the damage. There are other occasions where basic notions could have been treated more profoundly without adding to the difficulty. Generally speaking, what I personally like best about the book are the passages intended to get the student interested and the passages giving easy but illuminating examples of mathematics at work; what I like somewhat less are the passages concerned with the explanation of fundamental concepts. This is not necessarily a drawback: it leaves something for the course instructor to contribute. After all, I presume the book is designed for use as a class text rather than for self-teaching. For the same reason, some errors, omissions and ambiguities, which would otherwise be serious, are just minor irritations; they (and I noticed about a dozen) are easily spotted and dealt with by the lecturer.

On the whole, I think the book can be the basis of a most stimulating course. It is also very handsomely produced: clear type in
black and red on good quality paper, pleasing diagrams, pink and grey shading to distinguish examples and problems from the main text. It is good value for students and teachers of the kind of course for which the book is designed.

A. Zulauf


The reviewer has pleasant recollections of teaching advanced calculus courses from the first edition of this text. One of its strongest features is its popularity with students. As Professor Taylor wrote in his preface to the first edition, this book grew out of his experience in teaching advanced calculus over more than a dozen years, and every part 'has been planned to make the whole an effective instrument for imparting the fundamental principles and methods of analysis to students at the advanced calculus level. The book is aimed at the student reader . . . .'.

The first four chapters (125 pages) deal with fundamentals of elementary calculus, the real number system, continuous functions (e.g. the attainment of extreme values on closed bounded intervals and the intermediate value theorem), and extensions of the law of the mean (L'Hospital's rule, etc.). The inevitable logical inadequacies of a first course in calculus are recognised, and the student is helped to understand and appreciate rigorous analysis to the extent that it is needed in advanced calculus. Extensive collections of exercises are provided. For instance, the text includes a careful page-long discussion of the function $f$ given by $f(x) = x^2 \sin(1/x)$ if $x \neq 0$ and $f(0) = 0$, and properties of $g$ and $h$ given by $g(x) = x^3 \sin(1/x)$ and $h(x) = x^n \sin(1/x^2)$ if $x \neq 0$, and $g(0) = h(0) = 0$, are treated in the exercises.
The next eleven chapters (414 pages) are mainly concerned with techniques of the differential and integral calculus of functions of several variables.

This part of the book begins with a chapter on functions of several variables in general, with explicit attention being given to topics such as regions of definition of such functions, and modes of representing them. Many good students complete first degrees in mathematics with only a shaky knowledge of the elements of partial differentiation, but this should not happen to students who have been exposed to this book, which includes a long chapter designed to ensure that the student acquires facility in working with partial derivatives before being introduced to more theoretical topics. Under suitable differentiability assumptions on \( u = f(x,y) \), the partial derivative of \( f \) with respect to \( x \) is denoted by \( \partial u/\partial x \), \( \partial f/\partial x \), or \( f_1(x,y) \); the weaknesses of the first two of these notations are pointed out, but the text ensures that the student develops facility in working with all three. Topics covered in this chapter include derivatives of composite functions (including higher order derivatives), an introduction to differentials, and extremal problems including problems with constraints and Lagrange's multipliers. Succeeding chapters treat theorems about differentials and partial differentiation, implicit function theorems, transformations of co-ordinates and properties of Jacobians, and the elements of vector analysis (grad, div and curl).

At this point two new chapters have been added in the second edition. These chapters treat abstract vector spaces over \( \mathbb{R} \), linear transformations from \( \mathbb{R}^n \) to \( \mathbb{R}^m \), normed spaces over \( \mathbb{R} \), norms of linear transformations, invertible operators on \( \mathbb{R}^n \), differential calculus of functions from \( \mathbb{R}^n \) to \( \mathbb{R}^m \) (differentials and continuously differentiable transformations, inverse and implicit function theorems), and the equivalence of norms in finite dimensional
vector spaces. These new chapters do not seem to be properly integrated with the rest of the book. For instance, immediately after these chapters the abstract approach is abandoned and the authors revert to functions of two or three variables. Thus on p. 325 there is a reference back to pp. 135-6 for the attainment of a maximum value by a continuous real valued function on a closed bounded subset of $\mathbb{R}^n$. However, the theorems on pp. 135-6 are only for functions of two or three variables, and are not proved until p. 558 and still only at this level of generality. It would perhaps have been more appropriate if the material in the new chapters had been presented as an appendix at the end of the book.

The treatment of techniques of multivariable calculus concludes with chapters on multiple integrals (discussion of questions of integrability being deferred until later in the book), elementary differential geometry of curves and surfaces (arc length, tangent vector, normal and binormal, curvature and torsion, surface area, envelopes), and line and surface integrals (including transformations of multiple integrals and Green's and Stokes's theorems).

The final seven chapters (210 pages) contain a deeper study of the fundamentals of calculus. This part of the book comprises a chapter on point-set theory in one, two or three dimensions (Bolzano-Weierstrass theorem, Cauchy's criterion, Heine-Borel theorem), a further chapter on continuous functions of up to three variables (attainment of extreme values on closed bounded sets, uniform continuity, and the intermediate value theorem), and chapters on Riemann integration theory (including multiple and iterated integrals), infinite series, uniform convergence, power series and improper integrals (including the gamma and beta functions). The final two chapters of the first edition, on complex functions and Fourier series and integrals, have been dropped from the second edition.
This text should be suitable for a wide variety of classes. The new material in the second edition will probably be of value mainly for individual reading by better students, but the material carried over from the first edition should ensure that this work remains a very effective teaching aid.

In the preface the authors acknowledge the insight they have gained from the questions and comments of their students, and express the hope that other students and other teachers will find that this book opens the doors to understanding and enjoyment. The reviewer believes that this hope will be realised.

J. A. Kalman


This book has been designed to introduce the subject of homological algebra, assuming only a basic background in the theory of rings and modules. It will prove most suitable for first year graduate students wishing to learn the techniques of the subject and some of the more interesting results it yields.

One of the main features of the book is that the author avoids the often complicated machinery used to introduce Ext functors of arbitrary degree and instead concentrates on a simple construction of the functor

\[ \text{Ext}^1(A,B) \]

At the same time, however, a good introduction to the theory of homological dimension of modules and rings is presented.

The first three chapters are devoted to introducing the language and tools of homological algebra, in particular functors of modules -
the hom functor being the most important. Projective modules, injective modules, injective envelopes, and the functor $\text{Ext}^1$ are also studied.

Chapters 4, 5 and 6 deal with applications. Polynomial rings and the equivalence between a ring and its matrix rings are covered in chapter 4 while chapter 5 investigates Artinian and Noetherian rings and duality for modules over these rings. In chapter 6 the author considers more recent work in treating projective covers and local rings - some hitherto unpublished material appears in this, the last chapter.

A novel feature of the book is the large number of exercises appearing throughout the text and used in the development of the subject. The solutions of these are given at the end of the chapters. In conclusion this book is of the high standard one has come to expect from Professor Northcott and it will be very useful to the student or research worker requiring an introduction to homological algebra.

J. Clark


This edition of "The Theory of Partial Differential Equations" by Professor S. Mizohata is an English translation, including some improvements to the exposition suggested by the author, of the original Japanese text: *Henbibun-Hoteishiki Ron*, published in 1965. The name of the translator is not given.

The aim of this book is to give, using functional analysis, an introduction at the graduate level to the theory of partial differential equations. The prerequisite for the reading of the book is an elementary knowledge of Lebesgue integral and functional analysis.
The Distribution Theory of L. Schwartz is used throughout the book and the derivatives are always taken in the sense of this theory. A résumé of Distribution Theory is given in Ch. 2. $S$ and $(S)$, $S'$ and $(S')$ denote the same spaces, $S'$ being the dual of $S$.

The core of the book concerns the theory of linear (and mainly of second order) elliptic equations and hyperbolic equations. The $L^2$ theory is used.

For the study of hyperbolic equations with variable coefficients (Ch. 6), the method of Friedrichs for symmetric hyperbolic systems is used in preference to the method of Leray and Garding. Energy inequalities and the singular integral operators of Calderón-Zygmund are used. The weakly hyperbolic equations with variable coefficients are not discussed.

In Chapter 4, the author considers hyperbolic equations with constant coefficients. At the beginning of this chapter, after having given proofs for the Cauchy-Kowaleski and Holgrem theorems, the author discusses the local solvability of equations with $C^0$ and $C^\infty$-class coefficients and the continuity of solutions for the Cauchy problem.

In Chapter 7, the author is concerned with a semi-linear hyperbolic equation. An existence theorem for the local solution of the Cauchy problem is proved by using the Sobolev inequalities. It is shown that a priori estimates allow the construction of global solutions and the example of a semi-linear wave equation is given, which has a priori estimation.

Chapter 3 is consecrated to elliptic equations, in particular to the Dirichlet and Neumann problems. The Dirichlet problem is solved by defining Green's operator and using the Hilbert-Schmidt theory, which is explained in detail. Problems with other boundary conditions
are considered and the regularity of solutions is established.

Chapter 8 is on Green's functions and spectra discussed rather in a traditional fashion. An application of a Green's function is made to the behaviour of the solution of the Cauchy problem for a wave equation in an exterior domain in $\mathbb{R}^3$. Discrete spectra of Schrödinger's type operators are discussed.

Chapter 5 is devoted to Evolution problems. An existence theorem for the solution of the Cauchy problem is established which in its nature is entirely different from the Cauchy-Kowaleski existence theorem (Ch. 4). Analytic (or parabolic) semi-groups are discussed using the Hille-Yosida theorem and an existence theorem for a parabolic equation is established.

Chapter 1 deals with Fourier series and Fourier transforms in a rather dense fashion. To discuss the case of several variables, functional spaces used in Distribution Theory are introduced. Some would have preferred, for logical reasons, to integrate this chapter with chapter 2.

Each chapter can be read independently of the others, with some knowledge of the contents of chapter 2.

This book is a good one for somebody who wants to explore the main countries and products of the modern theory of partial differential equations.

For more recent results, one can have a look at the survey: *Linear Differential Operators* by L. Hörmander, Actes. Congrès Inter. Math., Nice, 1970, Tome 1, pp. 121-133 and the list of references he gives.

For the non linear case, one can consult the address of

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